One-dimensional Numerical Model of Cohesive Sediment Transport in Open Channel Flow

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ABSTRACT

Cohesive sediment transport remains a complicated subject that hydraulic engineers are frequently faced with in water-related engineering problems. This is primarily affected by the macroscopic aspects of water-sediment system characteristics. In this paper a 1-D mathematical model was developed to be employed in predicting the cohesive sediment transport under simultaneous conditions of erosion and deposition. This model is based on the convection-diffusion equation with proper source and sink terms and dispersion coefficient. The equation developed in the model has been solved by applying the finite volume approach. The model has been calibrated by employing the optimization technique using laboratory experimental data. For optimization, the transformed Powel's method has been employed. The data were collected in a flume of 10 m length, 0.30 m width and 0.45 m height. The applied discharges and concentrations were between 3 to 5lit/sec and 7 to 15 lit sec^{-1}, respectively. The performance of this model has been assessed using two data sets: a set obtained in this study, and another provided by other researchers. The model shows good agreement with both data sets. The results obtained suggest that the deposition and erosion are functions of flow concentration, flow depth and shear stress exerted on bed.

Keywords: Cohesive sediment, Convection-dispersion equation, Deposition, Erosion.

INTRODUCTION

Dealing with cohesive sediment transport remains a recurrent problem in water engineering as well as in many other related disciplines. This is especially important in engineering projects that involve riverbank stability, onshore sediment transport, scouring around bridge piers, and as well water quality problems. At present, no general analytical theory for cohesive sediment resuspension is available. As such empirically based field and laboratory experiments are needed. This stems, primarily, from the fact that cohesive sediment transport is governed not only by hydrodynamic forces but also by electrochemical ones as well. Due to the continuous complex process of fine cohesive sediment under different cycles of erosion, advection, turbulent and molecular diffusion, dispersion, flocculation, deposition, and consolidation the prediction of cohesive sediment movements is a complicated task. These processes are time dependent, nonlinear and multiphase (Scarlatos and Li, 1997). The simulation of any of the erosion stages cycle is a difficult task. Cohesive sediment may experience different conditions as moving, being deposited or suspended and sometimes being compacted. Therefore, in simulation the different stages of erosion-deposition process have to be put into consideration. The erosion or deposition equations for cohesive sediments are basically considered in the sink/source term of the convection-diffusion equation. Different researchers

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defined the sink/source terms in different ways, then using calibration, they proposed different erosion and deposition equations. In this regard, Krone (1962) considered erosion rate as a function of sediment falling velocity, flow sediment concentration, and the ratio of average bed shear stress to critical shear stress. The relationship is widely used in different investigations. Scarlatos (1981) proposed a cohesive sediment transport relationship. He considered erosion as a function of flow velocity, hydraulic radius and bed roughness coefficient. Reinaldo *et al.* (1999) proposed the erosion term as a function of flow average velocity, flow depth and the ratio of flow shear stress to the erosion critical shear stress. He also defined the deposition term as a function of flow concentration and falling velocity of fine sediments and solved the continuity and convection-diffusion equations. Li (1997) employed a similar equation in a 1-D model simulation for Qiantang River in China. Krone (1999) introduced a relation for erosion evaluation as a function of the ratio of average shear stress embedded on bed to critical shear stress. Roberts *et al.* (1998) investigated the effect of sediment size and density related erosion rate. His results show that erosion rate is drastically a function of sediment density and size.

To determine the turbulence dispersion coefficient, different researchers have conducted different investigations. For instance, Reidar and Olsen (2002) defined it as a function of flow sediment concentration and flow velocity. Another research showed the coefficient as a function of eddy viscosity and Schmidt number where the relation was based on real field data (Lin and Falconer, 1996). Hayter (1995) employed a two dimensional model in his cohesive sediment transport investigation. In his simulation, he considered the dispersion coefficient as a function of average velocity, flow depth and shear velocity. He also, used the sink/source term to define the erosion and deposition relations. The calibration of his model has been conducted by employing

experimental data collected for the research work. The different erosion optimum parameter values, deposition and dispersion relations, have been determined within the convection-dispersion equation using a numerical method and optimization technique.

In this research, the 1-D convection-dispersion equation has been numerically solved by the finite volume approach as an implicit scheme. Solving the equation gives the spatial and temporal cohesive sediment concentration values along the flow channel route. Thus, by using the sediment continuity equation, the depths of the channel bed along the channel are obtained. Parameters involved in the erosion-deposition and dispersion processes are determined through an optimization technique.

Modeling

To solve the convection-dispersion equation for cohesive sediment concentration numerically, the erosion or deposition relations are treated as a sink/source term, and the dispersion coefficient is defined by the dispersion relationship in an implicit solution scheme context. The convection-dispersion equation and the finite volume descritised form of it are as (Patankar, 1979):

$$\frac{\partial c}{\partial t} + \frac{\partial(u.c)}{\partial x} = \frac{\partial}{\partial x}(D\frac{\partial c}{\partial x}) + S \qquad (1$$

$$(\frac{-\Delta t}{\rho.\Delta x}a_W)C_{i-1}^{n+1} + (1+\frac{\Delta t}{\Delta x}a_P)C_i^{n+1} + \qquad (2$$

$$(\frac{-\Delta t}{\Delta x}a_E)C_{i+1}^{n+1} = C_i^n + \Delta t.S_i^{n+1}$$

$$a_E = \Delta x.D_e A(|P|) + \lfloor -u_e,0 \rfloor \qquad (3$$

$$a_W = \Delta x.D_w A(|P_w|) + \lfloor u_w,0 \rfloor \qquad (4$$

$$a_P = a_E + a_W + (u_e - u_w) \qquad (5$$

$$A(P) = \frac{|P|}{Exp(P)-1} \qquad (6$$

$$P = \frac{u.\Delta x}{D} \qquad (7$$

Note that the notation [*A, B*] refers to the maximum value between *A* and *B*. The deposition and erosion rates can be formulated as:

$$f(S_e, \rho, \rho_s, g, h, v, t, \mu, D_s, C, S) = 0 \qquad (8$$

$$f(\rho, \rho_s, S_d, t, \mu, D_e, v, C, S_0, h, g) = 0 \qquad (9$$

Using the Buckingham Π theorem, the deposition and erosion rates can be expressed by the following dimensionless variables:

$$S = S_e - S_d \qquad (10$$

$$S_e = A_1 (\frac{\rho v}{h})(\frac{v.t}{h})^{B_1} (\frac{\rho v h}{\mu})^{B_2} (\frac{\tau_0 - \tau_e}{\rho v^2})^{B_3}$$

$$.(\frac{v^2}{gh})^{B_4} (\frac{h}{D_s})^{B_5} (C_0)^{B_6} (\frac{\rho}{\rho_S})^{B_7} \qquad (11$$

$$S_d = A_2 (\frac{\rho v}{h})(\frac{v.t}{h})^{B_8} (\frac{\rho v h}{\mu})^{B_9} (1 - \frac{\tau_0}{\tau_d})^{B_{10}}$$

$$(C_0)^{B_{11}} .(\frac{v}{\omega_s})^{B_{12}} (\frac{v^2}{gh})^{B_{13}} (\frac{\rho}{\rho_s})^{B_{14}} (\frac{h}{D_s})^{B_{15}} \qquad (12$$

$$D = A_3 (vh)(\frac{v}{u_*})^{B_{16}} (\frac{\rho v h}{\mu})^{B_{17}} \qquad (13$$

$$\omega_s = C_1 D_s^{C_2} \qquad (14$$

where ρ_s is the dry sediment density, h the flow depth, v is the flow velocity, μ the dynamic viscosity, D_s is the sediment particle diameter, S_0 the channel slope, ω_s is the sediment falling velocity, ρ the fluid density, g is the acceleration due to gravity, C the suspended load concentration, τ_e, τ_d and τ_0 are the erosion and deposition critical shear stresses and average shear stress on bed respectively, D is turbulent dispersion coefficient, u_* the flow shear velocity, S_d is the sediment deposition rate, S_e the sediment erosion rate and $A_1, A_2, A_3, B_1, ..., B_{17}, C_1, C_2$ are the unknown coefficients to be determined in the calibration process. Equation 1 has been solved by using finite volume method.

By solving the differential Equation (1), the sediment concentrations can be determined for different time and space increments. Subsequently the sediment discharge can be calculated from the concentration distribution, and by applying the sediment continuity equation, the sediment thickness for different sections can be computed. The sediment continuity equation is:

$$B\frac{dZ}{dt} + \frac{1}{1-n}\frac{dQ_S}{dx} = 0 \qquad (15$$

where *B* is the flow channel width, *Z* the bed sediment thickness, *t* is time, *n* is the sediment porosity, and Q_S the sediment discharge. Finally, the unknown coefficients can be obtained by employing the experimental data along with an optimization technique. For this purpose, the modified Powel's method was employed. This method has been derived from the original Powell method. The method is generally used when differentiations of functions are difficult or impossible. In this study optimization is based on minimizing the following objective function, using a computer program code:

$$\min\left[\left(\frac{\sum(Z_o - Z_c)^2}{N}\right)^{0.5}\right] \qquad (16$$

where *N* refers to the number of data points, Z_o is the observed bed thickness and Z_c is the calculated bed thickness.

Experiments

Flow and cohesive sediment data were collected by conducting a set of erosion and deposition experiments, a total of 24 runs, using 2 sediment diameters, 2 flow rates, and 3 flow sediment concentrations. Considering that shear stresses can be lower or higher than τ_c and τ_d the slopes selected for the experiments were $S_0 = 0.00002$ for deposition and $S_0 = 0.00005$ for erosion runs. The data were collected in a flume of 10 m length, 0.30 m width and 0.45 m height. Due to the limitation of the channel, the selected flow rates were 3 lit sec^{-1} and 5 lit sec^{-1}. The

discharge values, sediment particle size and concentration values were set to 3 and 5 lit sec^{-1}, 21 and 35µm, and 6, 9, and 12 g lit^{-1} respectively. The durations for the deposition and erosion experiments were 4 and 1 hour (s) respectively.

DISCUSSION

The required data for calibrating and verifying the model were collected through conducting laboratory experiments. The data were collected in a flume of 10 m length, 0.30 m width and 0.45 m height. The applied discharge rates and concentrations were 3 to 5 lit sec^{-1}and 7 to 15 g lit^{-1} respectively. Fifty percent of the performed tests were used for model calibration. Mean absolute and relative errors were 0.35 mm and 8% respectively. The correlation coefficient between observed and calculated values (R^2) was 0.956 for a confidence level of 99%. From among various alternative functions, the one with exponents equal to unity for the source term in the convection-dispersion equation was selected.

Due to the negligible error of the parameter, $\dfrac{\rho}{\rho_s}$ and the Froude number were omitted from the source term. Froude number ranged between 0.064 and 0.071 throughout the study, and the omitting of the parameters kept the correlation coefficient as high as 0.972. For further simplification the parameters $\dfrac{h}{D_s}$, Reynolds number and Strouhal number were also omitted. As a result the final correlation coefficient and relative error were 0.918 and 6.7%, respectively. The ultimate relationships obtained for the different terms are as follows:

$$S_d = 0.00164(\frac{\rho.v}{h})(1 - \frac{\tau_0}{\tau_d})^{0.145}$$

$$(C_0)^{1.256}(\frac{v}{\omega_s})^{0.045} \tag{17}$$

$$D = 148.7(vh)(\frac{v}{u_*})^{-1.013} \tag{18}$$

$$(\frac{\rho.v.h}{\mu})^{0.345}$$

$$\omega_s = 0.0045 D_s^{0.058} \tag{19}$$

In deposition process, flocculation increases drastically (Teisson, 1992); therefore the falling velocity of sediment particles was assumed to be a nonlinear function of sediment particle diameter. Similar to the source term, among the various tested functions the one with exponents equal to unity for the sink term was employed. Omitting the Reynolds and Strouhal numbers led to correlation coefficient and relative error values of 0.805 and 6% respectively. As a result the final relations for the erosion rate are as belows:

$$S_e = 0.00013(\frac{\rho v}{h})(\frac{v^2}{g.h})^{0.039}(\frac{h}{D_s})^{0.118}$$

$$(C_0)^{0.303}(\frac{\tau_0 - \tau_e}{\rho.v^2})^{0.622} \tag{20}$$

$$D = 110.4(vh)(\frac{v}{u_*})^{-0.677}(\frac{\rho.v.h}{\mu})^{0.294} \tag{21}$$

In the calibration procedure, the turbulence dispersion coefficient was selected by employing the optimization technique in a way that a minimum error could be encountered in both sedimentation and erosion models. The final function was determined as:

$$D = 130(vh)(\frac{v}{u_*})^{-0.84}(\frac{\rho vh}{\mu})^{0.32} \tag{22}$$

RESULTS

Table 1 shows the results found based on the proposed model.

The calibration and verification results are presented in Figure 1a-d.

For more validation, experiment data acquired by other researchers were employed. In this regard, the Scarlatos and Li (1997) deposition experiment data were

Table 1. Deposition and erosion relative error, and correlation coefficient.

Model	Relative error (%)	Correlation coefficient (R^2)
Final deposition model	15.2	0.87
Final erosion model	4.1	0.83

used. The relative error of Scarlatos and Li compared to the present model were 94 and 56 percent respectively. Table 2 shows the results obtained from the present study and those obtained by Scarlatos and Li. The present model shows better results as compared to the Scarlatos ones.

CONCLUSIONS

Throughout this research, one-dimensional convection-diffusion equation has been solved numerically using the finite volume method. Laboratory experiments have been conducted for validating the model. Other researchers' experimental data have also been taken advantage of for further evaluation of the model. A comparison of the observed and calculated deposited and eroded depths implies good accuracy and precision of the proposed model. Also the present model could predict bed thicknesses more precisely than Scarlatos model.
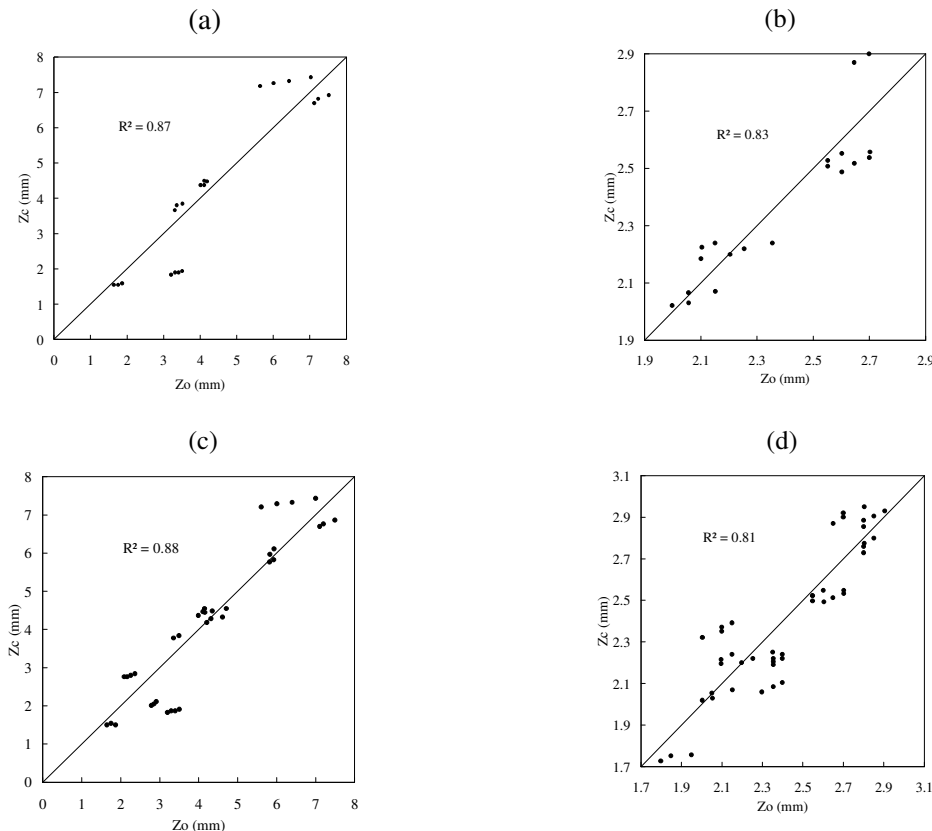

Figure 1. Observed and calculated bed thicknesses in: verification of the deposition model (a), values in verification of the erosion model (b), the sedimentation model calibration for the entire(c) and values in the erosion model calibration for the entire data(d).

One important advantage of this model is the use of basic fluid dynamic equation (convection-diffusion) in predicting sediment transport behavior, making it more precise than the other similar models.

Nomenclature

$A_1, A_2, A_3, B_1, ..., B_{17}, C_1, C_2$

  Unknown coefficients to be determined in the calibration process

ρ_s Dry sediment density

h Flow depth

v Flow velocity

μ Dynamic viscosity

D_s Sediment particle diameter

S_0 Channel slope

ω_s Sediment falling velocity

ρ Fluid density

g Acceleration due to gravity

C Suspended load concentration

τ_e Erosion critical shear stress

τ_d Deposition critical shear stress

τ_0 Average shear stress

D Turbulent dispersion coefficient

u_* Flow shear velocity

S_d Sediment deposition rate

S_e Sediment erosion rate

B Channel width

Z Bed sediment thickness

t Time

n Sediment porosity

Q_S Sediment discharge

N Number of data points

Z_o Observed bed thickness

Z_c Calculated bed thickness

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مدل یک بعدی برای شبیه سازی رسوبگذاری رسوبات چسبیده در مجاری رو باز

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چکیده

توصیف جزئی رفتار رسوبات ریزدانه چسبنده یک موضوع بسیار پیچیده می‌باشد و شبیه‌سازی حرکت رسوبات چسبنده بطور شدیدی تحت تاثیر خواص ماکروسکوپی از سیستم آب – رسوب می‌باشد. در این تحقیق با تهیه یک مدل ریاضی که اساس آن معادله انتقال جرم می‌باشد و با پیشنهاد معادلات رسوبگذاری, فرسایش و ضریب انتشار در جمله منبع این معادله، معادله انتقال جرم با روش حجم کنترل حل شده و با استفاده از داده‌های آزمایشگاهی، ضرائب بهینه معادله پیشنهادی با اعمال بهینه‌سازی ریاضی واسنجی شده است. برای بهینه‌سازی از روش پاول استفاده به عمل آمده است. آزمایشات در یک فلوم آزمایشگاهی به عرض ۳۰ سانتیمتر، دبی جریان بین ۳ تا ۵ لیتر در ثانیه و غلظت های ۸ الی ۱۵ گرم در لیتر انجام شده است. در مدل پیشنهادی مشخص شد که مقدار فرسایش و رسوبگذاری شدیداً تحت تاثیر پارامترهای غلظت جریان، عمق جریان و تنش برشی متوسط جریان می‌باشد. همچنین این مدل از هماهنگی بهتری با داده‌های آزمایشگاهی در مقایسه با اکثر مدل های پیشنهادی برخوردار بود.