

Regression Models for Milk Yield and Intake in Dairy Ration Optimization: An Observational Study at the Chichak-Sud Commercial Dairy Farm

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ABSTRACT

Predictive models may help dairy farms allocate feed, but their use requires biological and economic checks. This observational study compared five regression approaches and linked animal-level predictions to a linear programming scenario for ration formulation. The dataset comprised 2,911 cow-month records from 82 Holstein cows at a commercial farm in Azerbaijan during 2023–2025. Predictors described body size, lactation stage, parity, body condition and dietary nutrient composition. Ordinary least squares, ridge, least absolute shrinkage and selection operator, elastic net and second-degree polynomial regression were compared using training and test sets separated by cow. Elastic net had the lowest reported test errors: root mean squared errors of 2.36 liters per day for milk yield and 1.18 kilograms of dry matter per day for intake, with coefficients of determination of 0.84 and 0.80, respectively. The optimization summary reported feed costs of 5.07 and 4.39 Azerbaijani manat per cow per day for standard and candidate rations, with predicted milk volumes of 22.8 and 22.6 liters per day. However, a consistency audit found that the reported ingredient quantities and composition matrix did not reproduce the ration nutrient totals. The economic estimates therefore remain provisional. The framework supports further evaluation of ration decisions; reconciliation of the input matrices, additional nonlinear and seasonal comparisons, milk-composition data, external validation and a controlled feeding trial are required before implementation.

Keywords: Dairy cows, Dry matter intake, Elastic net, Linear programming, Ration optimization.

INTRODUCTION

Ration formulation should match nutrient supply to dairy-cow requirements and control feed expenditure, following nutrient guidance from the National Research Council (NRC, 2001) and farm-level economic assessment (Bach et al., 2020). Herd averages can obscure

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between-animal variation, making nutritional resource allocation important for farm management (Bellingeri et al., 2020).

The State Statistical Committee of the Republic of Azerbaijan (SSC) reported milk production of 2.2985 and 2.3059 million metric tons in 2024 and 2025. **National annual milk yield per cow and buffalo averaged 1,834 and 1,857 kg, respectively (SSC, 2025, 2026). These heterogeneous national averages are not breed-specific estimates for intensive Holstein herds and are not directly comparable with the present farm's daily lactation records.**

Between-cow variation in lactation curves is associated with parity, lactation stage and management (Tekerli et al., 2000; Ghavi Hossein-Zadeh, 2014). A common ration can consequently supply different amounts of nutrients relative to individual requirements. Optimization studies have examined allocation across a herd and diets for individual or grouped cows (Bellingeri et al., 2020; Campos et al., 2023).

Mathematical programming for dairy diets is established. Tozer and Stokes (2001) combined feed cost and nutrient-excretion objectives; Bellingeri et al. (2020) linked crop planning with nutritional allocation; and Campos et al. (2023) optimized individual and grouped diets within a physiological nutrient framework. These studies establish the method's potential, rather than demonstrating that a prediction model transfers unchanged to another farm.

The practical question here is whether routinely recorded animal and feed variables can inform a ration scenario under one Azerbaijani farm's feed resources and prices. The contribution is a local application and assessment of a prediction-to-formulation workflow, rather than a new regression algorithm or a demonstrated feeding treatment. Its limitations include incomplete seasonal and milk-composition adjustment and the need to reconcile the ration inputs.

The objective was to compare five regression models for daily milk yield and dry matter intake (DMI), use the selected model's predictions to define a least-cost ration scenario, and assess its reported feed cost and income over feed cost (IOFC).

MATERIALS AND METHODS

Study Site Design and Animals

The study used longitudinal observational records from the Chichak-Sud commercial dairy farm in Turkmen village, Barda District, Karabakh economic region, Azerbaijan. The

observation window covered January 2023–December 2025. The dataset represented 82 Holstein cows with repeated monthly measurements; no randomized feeding treatment was assigned.

Cows were milked twice daily and received the farm’s total mixed ration (TMR). Ethical approval for conducting the observational study was granted by the Ethics Committee of Azerbaijan State Agricultural University (Protocol No. 2022/14). Predictions describe associations under the recorded management conditions and do not estimate the causal effect of a ration change.

Animal and Milk Measurements

Monthly records included body weight (BW, kg), age (months), days in milk (DIM, days), parity (lactation number), body condition score (BCS) and milk yield (L/day). Body weight was measured using a livestock scale. Body condition score was assessed on a 1–5 scale following Ferguson et al. (1994). Calibrated in-line flow meters measured milk volume. Records were restricted to DIM 5–305; the first four days after calving were excluded.

Cow-month milk fat and protein measurements were not available in the analytical dataset. The response therefore remains milk volume, rather than energy-corrected milk (ECM). Volumetric accuracy cannot establish accuracy for milk-energy output, and the results do not demonstrate equal milk composition or equal energy yield between ration scenarios.

Feed Composition Prices and Intake

Monthly samples were collected separately from corn silage, alfalfa hay, wheat straw, compound feed and mineral–vitamin premix. Ingredient records supplied dry matter (DM) fraction, crude protein (CP, g/kg DM), metabolizable energy (ME, MJ/kg DM), neutral detergent fiber (NDF, % DM), acid detergent fiber (ADF, % DM), calcium (Ca, % DM) and phosphorus (P, % DM). The average ingredient matrix used in the base scenario is shown in Table 1.

Prices were recorded in Azerbaijani manat (AZN) per kilogram of fresh matter (FM). Prices and ingredient quantities must use the same basis; nutrient supply was calculated from FM quantity multiplied by the ingredient’s DM fraction. The local currency was retained because both feed purchases and the milk-price scenarios use AZN.

98 **Table 1.** Average ingredient composition and price coefficients used in the base scenario.

Ingredient	DM fraction	CP	ME	NDF	ADF	Ca	P	Price
Corn silage	0.34	82	10.8	45	26	0.28	0.22	0.087
Alfalfa hay	0.88	175	9.4	42	31	1.35	0.25	0.174
Wheat straw	0.90	42	6.8	72	45	0.32	0.09	0.104
Compound feed	0.89	210	12.8	19	8	0.75	0.55	0.391
Mineral–vitamin premix	0.95	0	0.0	0	0	18.00	8.00	1.085

99 DM is a fraction of FM; CP and ME are expressed per kg DM, and fiber and mineral concentrations are
100 percentages of DM. Monthly composition and prices must be retained with their sampling dates when reproducing
101 or updating the scenario.
102

103 Individual-cow DMI was calculated from electronically weighed feed-bin visits linked to
104 radio-frequency identification (RFID). A shared TMR mixture does not imply a permanently
105 assigned bin for every cow. For each recorded visit, fresh intake was the difference between
106 feed delivered and feed remaining, subject to technical checks for spillage and refusals. Valid
107 daily fresh intake was multiplied by the monthly laboratory TMR DM fraction and averaged
108 within cow-month.

109 Weighing modules were checked weekly with standard weights. Days with missing RFID
110 readings, equipment failure, incomplete bin entry, implausible refusals or acute feed refusal
111 recorded by veterinary staff were excluded. Independent verification requires the feeder make
112 and model, access-control protocol, visit logs, calibration records and valid-day counts.
113 Laboratory assay methods and the derivation of the feed ME values also require documentation
114 in the reproducibility package.
115

116 **Model Comparison and Validation**

117 The initial 2,952 records were reduced to 2,911 after exclusion of 18 records with more
118 than 5% missing values and 23 records linked to illness, feed refusal or technical error. The
119 main comparison separated cows between training (65 cows; 2,329 records) and testing (17
120 cows; 582 records). Five-fold cross-validation grouped by cow selected tuning parameters. A
121 2023–2024 training period with a 2025 holdout provided an additional temporal check.
122 Preprocessing and resampling details are given in Appendix A.

123 The predictors were BW, age, DIM, parity, BCS, CP, ME, NDF and ADF. Multiple linear
124 regression (MLR) was the reference model; ridge, least absolute shrinkage and selection
125 operator (LASSO) and elastic net regressions assessed penalized alternatives (Tibshirani, 1996;
126 Zou and Hastie, 2005). Python 3.11, scikit-learn and SciPy were used (Pedregosa et al., 2011;
127 Virtanen et al., 2020).

The fifth model included quadratic terms for BW, DIM, BCS and ME and six pairwise interactions among them. It therefore permits a nonlinear response to these predictors while remaining linear in its coefficients. This differs from intrinsically nonlinear lactation models such as Wood's curve (Wood, 1967; Ghavi Hosseini-Zadeh, 2014). Third- and higher-degree polynomials, spline models and nonlinear lactation functions were not evaluated. Model ranking is restricted to the five tested approaches.

Calendar month was not included as an explicit predictor. The 36-month window covers three annual cycles, but a temporal holdout cannot isolate season from feed batches or management changes. The results therefore remain conditional on this specification; calendar-month or cyclic seasonal terms and weather-linked validation are needed to quantify seasonal sensitivity.

Performance was summarized by the coefficient of determination (R^2), root mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). Cow-clustered bootstrap intervals described test-set uncertainty. Model selection considered prediction error, stability and operational simplicity; it did not test whether a predicted ration would be consumed or sustain production.

Ration Formulation and Economic Scenario

Elastic net predictions provided milk-yield estimates and an individual DMI target for each cow-month. The linear program minimized ingredient cost within $\pm 5\%$ of predicted DMI. Nutrient profiles from the candidate solutions were subsequently entered into the milk-yield model. The reported 22.8 and 22.6 L/day values are model predictions under two profiles, rather than milk yields observed after feeding the candidate ration.

The five decision variables were daily FM quantities of the ingredients in Table 1. Ingredient bounds were corn silage 10–18, alfalfa hay 2.5–6, wheat straw 0.5–2, compound feed 3.5–8 and premix 0.10–0.20 kg FM/day. Moisture changes alter nutrient supply per kg FM. The base matrix used average composition; monthly DM and nutrient variability were not propagated into the reported feasibility or economic summaries.

The scenario specified CP ≥ 160 g/kg DM, NDF $\geq 28\%$ DM, ADF $\geq 18\%$ DM, Ca 0.60–0.80% DM and P 0.35–0.45% DM. These are study formulation screens, not universal requirements for every cow. National Research Council (NRC, 2001) guidance informed the original simplified energy calculation; the newer National Academies of Sciences, Engineering, and Medicine (NASEM, 2021) framework provides a basis for reassessment.

Neither the full physiological model nor total digestible nutrient or digestible crude protein calculations were implemented.

Metabolizable energy supply was intended to exceed the simplified maintenance-plus-lactation requirement by 5%. The individual requirement equations, milk-energy assumptions and efficiencies converting energy between systems need documentation before the ≥ 198 MJ/day scenario screen in Table 7 can be accepted. CP is not equivalent to metabolizable protein, and NDF and ADF alone do not establish adequate physically effective fiber or rumen function.

Income over feed cost was calculated as milk price $\times$ predicted milk yield – daily feed cost (Bach et al., 2020). Milk prices of 0.60, 0.80 and 1.00 AZN/L were scenario inputs. Ingredient-price sensitivity used $\pm 10\%$ and $\pm 20\%$ shocks with re-optimization. These outputs describe conditional decisions; they are not treatment effects or estimates of total farm profit.

RESULTS

Descriptive Statistics and Model Performance

Descriptive statistics are presented in Table 2; variability is expressed as standard deviation (SD). Mean milk yield was 22.8 ± 6.1 L/day and mean DMI was 18.9 ± 3.4 kg DM/day. Figure 1 displays the reported Pearson correlations: DIM with milk yield (-0.70), BW with DMI (0.73), ME with milk yield (0.56) and BCS with milk yield (-0.39). These are descriptive pooled associations across repeated cow-month records.

Table 2. Descriptive statistics for 2,911 cow-month records.

Variable	Unit	Mean $\pm$ SD	Minimum	Maximum
Body weight (BW)	kg	548 ± 52	415	668
Age	months	51 ± 15	24	90
Days in milk (DIM)	days	154 ± 87	5	305
Parity	lactation	2.3 ± 1.1	1	5
Body condition score (BCS)	1–5 scale	3.08 ± 0.39	2.0	4.5
Daily milk yield	L/day	22.8 ± 6.1	8.7	40.2
Dry matter intake (DMI)	kg DM/day	18.9 ± 3.4	11.0	28.1
Crude protein (CP)	g/kg DM	166 ± 13	138	202
Metabolizable energy (ME)	MJ/kg DM	11.1 ± 0.9	9.2	13.3
Neutral detergent fiber (NDF)	% DM	34.9 ± 3.3	27.0	42.5
Acid detergent fiber (ADF)	% DM	21.8 ± 2.4	16.8	28.1
TMR mean feed-cost coefficient	AZN/kg FM	0.29 ± 0.05	0.20	0.43

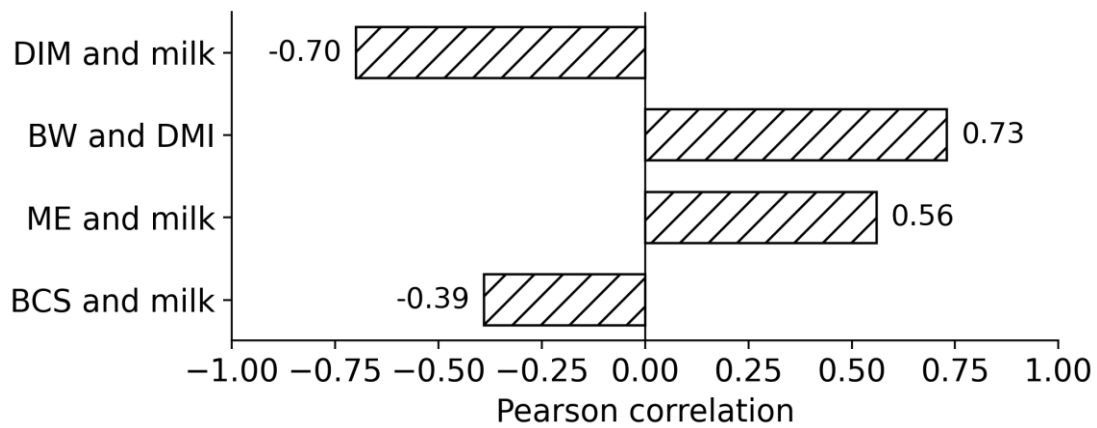


Fig. 1. Reported Pearson correlations for selected animal and diet variables. Values are descriptive associations across repeated cow-month records; the figure is not a full correlation matrix.

Milk-yield performance is summarized in Table 3. Elastic net had the lowest reported test RMSE, 2.36 L/day, compared with 2.47 L/day for the quadratic model; grouped cross-validation RMSE was 2.39 L/day. The quadratic comparison indicates that curvature was considered, but does not establish accuracy against untested nonlinear alternatives.

Table 3. Milk-yield prediction performance in the grouped test set of 17 cows and 582 records.

Model	R ²	RMSE	MAE	MAPE (%)
MLR	0.72	3.04	2.45	10.8
Ridge	0.77	2.78	2.24	9.9
LASSO	0.80	2.55	2.07	9.2
Quadratic polynomial	0.82	2.47	2.00	8.7
Elastic net	0.84	2.36	1.90	8.3

Selected elastic net parameters: $\alpha = 0.1$ and $\rho = 0.7$. RMSE and MAE are in L/day; MAPE is a percentage.

DMI results are shown in Table 4. Elastic net had RMSE 1.18 kg DM/day and R^2 0.80. Figure 2 compares model R^2 values for both responses. Table 5 reports bootstrap intervals and the 2025 holdout; the overlapping intervals for elastic net and polynomial regression do not establish a statistically decisive superiority of one model.

Table 4. DMI prediction performance in the grouped test set of 17 cows and 582 records.

Model	R ²	RMSE	MAE	MAPE (%)
MLR	0.69	1.46	1.12	6.4
Ridge	0.74	1.34	1.03	5.8
LASSO	0.77	1.25	0.96	5.3
Quadratic polynomial	0.79	1.20	0.92	5.0
Elastic net	0.80	1.18	0.89	4.8

Selected elastic net parameters: $\alpha = 0.1$ and $\rho = 0.6$. RMSE and MAE are in kg DM/day; MAPE is a percentage.

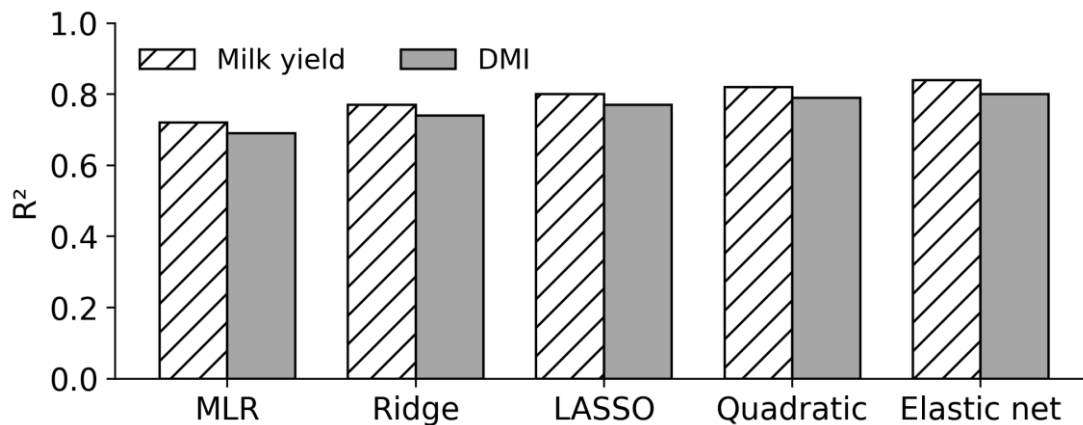


Fig. 2. Reported model R^2 values for milk yield and DMI in the grouped test set. The comparison covers the five evaluated models and does not establish performance against other nonlinear alternatives.

Table 5. Cow-clustered bootstrap intervals and 2025 temporal holdout performance.

Response	Model	R^2 95% interval	RMSE 95% interval	2025 R^2	2025 RMSE
Milk yield	Elastic net	0.84 (0.79-0.88)	2.36 (2.10-2.71)	0.82	2.49
Milk yield	Quadratic polynomial	0.82 (0.76-0.86)	2.47 (2.20-2.86)	0.80	2.61
DMI	Elastic net	0.80 (0.74-0.85)	1.18 (1.03-1.39)	0.78	1.26
DMI	Quadratic polynomial	0.79 (0.72-0.84)	1.20 (1.05-1.43)	0.76	1.31

Intervals are 95% percentile intervals from 2,000 bootstrap replicates. RMSE units are L/day for milk and kg DM/day for DMI.

Table 6 summarizes qualitative interpretation of standardized coefficients. These categories are not numerical importance scores and do not identify causal biological effects. DIM, BW and dietary energy were prominent predictive variables; interpretation depends on the other predictors and their correlations.

Table 6. Qualitative interpretation of standardized elastic net coefficients.

Variable	Milk model	DMI model	Biological context
DIM	High; negative	Moderate	Lactation stage
BW	Moderately positive high;	High; positive	Animal size and intake
ME	High; positive	Moderate	Dietary energy density
BCS	Moderately negative high;	Moderate	Body reserves
Parity	Moderate	Moderate	Maturity
CP	Moderate	Moderate	Protein supply
NDF	Low to moderate	Moderate	Fiber and intake
ADF	Low	Low to moderate	Fiber digestibility
Age	Low	Low	Age contribution

Reported Ration and Economic Scenarios

Table 7 presents the reported standard and candidate ration summaries; Figure 3 compares ingredient quantities. The reported feed-cost difference was 0.68 AZN/cow/day, or 13.4%, with a predicted milk-volume difference of -0.2 L/day. These figures remain provisional because the ingredient quantities and matrix do not reproduce several ration totals.

Table 7. Reported ration summaries requiring reconciliation with the ingredient matrix.

Parameter	Standard	Candidate	Study screen
Corn silage (kg FM/day)	12.4 ± 1.6	14.1 ± 1.7	-
Alfalfa hay (kg FM/day)	4.4 ± 0.7	3.9 ± 0.6	-
Wheat straw (kg FM/day)	1.1 ± 0.3	1.2 ± 0.4	-
Compound feed (kg FM/day)	6.5 ± 0.9	5.6 ± 0.8	-
Mineral–vitamin premix (kg FM/day)	0.16 ± 0.02	0.16 ± 0.02	-
Total DMI (kg DM/day)	18.9 ± 3.4	18.6 ± 3.2	Predicted DMI $\pm 5\%$
ME supply (MJ/day)	211 ± 19	207 ± 18	≥ 198
CP (g/kg DM)	166 ± 13	162 ± 11	≥ 160
NDF (% DM)	34.9 ± 3.3	36.2 ± 3.1	≥ 28
ADF (% DM)	21.8 ± 2.4	22.4 ± 2.1	≥ 18
Ca (% DM)	0.70 ± 0.06	0.68 ± 0.04	0.60–0.80
P (% DM)	0.40 ± 0.04	0.39 ± 0.03	0.35–0.45
Feed cost (AZN/cow/day)	5.07 ± 0.38	4.39 ± 0.33	-
Predicted milk yield (L/day)	22.8 ± 6.1	22.6 ± 5.9	-

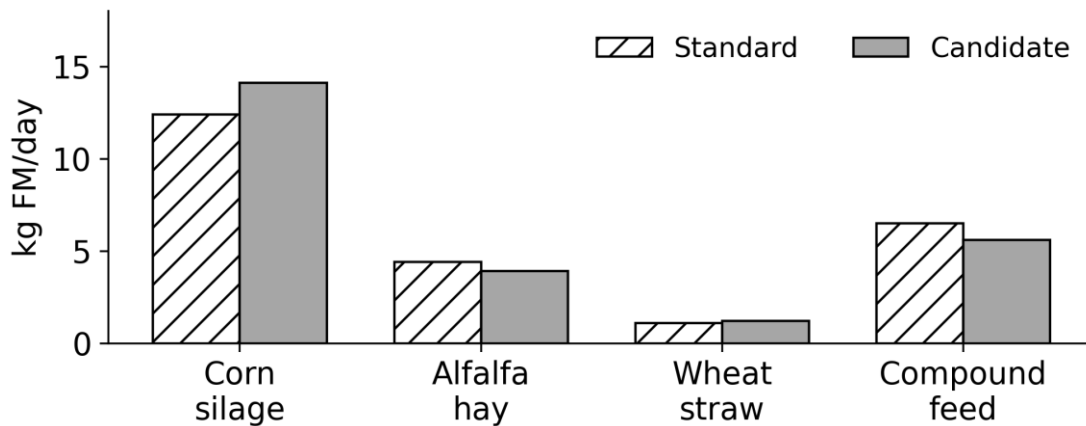


Fig. 3. Reported mean ingredient quantities for standard and candidate rations on an FM basis. The ingredient matrix and reported nutrient totals require reconciliation before nutritional feasibility can be concluded.

Using Table 1 coefficients with Table 7 mean FM quantities gives standard/candidate DM of 15.015/14.442 kg/day, ME of 162.710/155.175 MJ/day, CP of 151.831/144.420 g/kg DM and Ca of 0.919/0.886% DM. Corresponding matrix-based costs are 4.6739/4.3933 AZN/day. These disagree with the reported totals and would fail the stated energy, protein and Ca screens.

The aggregate means cannot verify individual solutions; claims that all nutrient constraints were satisfied have therefore been withdrawn pending reconciliation of the ingredient basis and solution files.

Table 8 shows IOFC calculated from the reported feed costs and predicted milk volumes. Its arithmetic yields advantages of 0.56, 0.52 and 0.48 AZN/cow/day across the three milk prices. The 3.40 AZN/L break-even price follows from 0.68/0.2. These calculations do not validate the underlying ration. Table 9 summarizes reported ingredient-price scenarios, whose feasibility status also requires the original solution diagnostics.

Table 8. Conditional IOFC calculations using the reported ration scenario inputs.

Milk price	Standard IOFC	Candidate IOFC	Daily difference	Annual difference 82 cows
0.60	8.61	9.17	0.56	16,761
0.80	13.17	13.69	0.52	15,564
1.00	17.73	18.21	0.48	14,366

Milk price: AZN/L; IOFC and daily difference: AZN/cow/day; annual difference: AZN.

IOFC excludes labor, housing, health, equipment and implementation costs. Annual differences assume 82 lactating cows for 365 days; they are scenario extrapolations, not observed annual profits.

Table 9. Reported ingredient-price sensitivity scenarios requiring solution-file verification.

Shock	Ingredient	Candidate cost	Reported savings	Status
-20%	Corn silage	4.15	0.92	Reported satisfied
+20%	Corn silage	4.64	0.43	Reported satisfied
-20%	Compound feed	3.95	1.12	Reported satisfied
+20%	Compound feed	4.83	0.24	Reported satisfied

Cost and savings are AZN/cow/day. Savings use the reported 5.07 AZN/day baseline; this fixed-baseline comparison is not a paired price-shock response of both rations. "Reported satisfied" records the supplied solver summary and does not resolve the matrix inconsistency.

DISCUSSION

The predictors represent biological attributes relevant to feeding: BW reflects animal size, DIM locates an observation within lactation, parity reflects maturity, and BCS describes body reserves (NRC, 2001; NASEM, 2021). Dietary energy, protein and fiber characterize nutrient supply and potential intake constraints. Their predictive associations do not demonstrate mechanisms, and age-parity and fiber-variable correlations can affect coefficient interpretation.

Penalization can stabilize regression with correlated predictors. Elastic net's small error advantage over the quadratic model should be interpreted alongside overlapping uncertainty intervals. The quadratic model permits curvature, but neither this comparison nor a negative pooled DIM correlation establishes the full rise, peak and decline of the lactation curve. Wood-

type functions, splines and higher-degree models require a common cow-grouped validation comparison before wider superiority can be claimed.

At the mean DMI of 18.9 kg/day, a $\pm 5\%$ intake band is approximately ± 0.945 kg/day, narrower than the reported DMI RMSE of 1.18 kg/day. RMSE is an aggregate error metric, not an individual prediction interval. The current bound therefore cannot guarantee sufficient intake for every cow. Wider or prediction-interval-based bounds need comparison, particularly for early-lactation or high-producing animals.

Lower compound-feed and higher silage inclusion may alter fermentable carbohydrate, physical fiber and palatability. A feasible mathematical solution would still require checks of metabolizable protein, starch, fiber effectiveness, mineral balance and feed availability. Here the matrix audit additionally prevents acceptance of the reported candidate as a nutritionally verified ration. Evidence that cows would consume it, maintain health or produce 22.6 L/day is unavailable.

Individual DMI measurement and individual ration delivery are separate capabilities. A TMR farm can use predictions to support nutritional grouping or supervised supplementation only where feed access and equipment permit. The proposed early-, mid- and late-lactation groups were implementation options, not tested treatments. Farm profitability alone does not establish the adequacy of every animal's ration, and IOFC does not measure total profit.

Limitations and Validation Needs

This single-farm observational dataset limits generalization and causal interpretation. Only 17 cows formed the main independent test sample. Calendar month, weather, changing management, feed availability and milk composition were not fully modeled. The reported DMI and laboratory procedures need auditable metadata. The ration matrix mismatch requires reconciliation before economic results can support a feeding recommendation. Independent herds and controlled trials should evaluate consumed DMI, ECM, BCS, health and actual feeding costs.

CONCLUSIONS

Among the five evaluated regressions, elastic net gave the lowest reported prediction errors for milk volume and DMI and was linked to a least-cost ration scenario. This supports investigation of a farm-level prediction-to-formulation workflow. The candidate ration and its economic advantage remain provisional because the supplied ingredient matrix does not

reproduce the reported nutrient totals. Biological adequacy, consumption and maintained milk production require reconciled inputs, further model comparisons and prospective validation.

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APPENDIX A

Statistical Specification and Reproducibility

Missing values were imputed by parity and lactation stage using training data only; standardization for penalized models was fitted within training folds. Records with $|z| > 3$ were screened against veterinary information. GroupKFold kept each cow in one-fold. The elastic net penalty strength was $\alpha \in \{0.001, 0.01, 0.1, 1, 10\}$, with L_1 mixing fraction $\rho \in \{0.1, 0.2, \dots, 0.9\}$. The quadratic model added BW^2 , DIM^2 , BCS^2 , ME^2 , $BW \times DIM$, $BW \times BCS$, $BW \times ME$, $DIM \times BCS$, $DIM \times ME$ and $BCS \times ME$ to the nine base predictors.

For uncertainty estimation, the 17 test-cow identifiers were sampled with replacement 2,000 times, retaining all monthly records of each sampled cow and their multiplicity (Efron and Tibshirani, 1993). The 2.5th and 97.5th percentiles defined the reported intervals. The temporal holdout assessed prediction for 2025, not generalization to an independent herd. No feeding-treatment significance test was inferred from these scenarios.

For the five ingredients, x_i denotes FM quantity, d_i the DM fraction, c_i the FM price and q_i a nutrient concentration. Equations [1]–[4] define daily cost C , DM amount D , the intake band and minimum nutrient and energy constraints. The inequality reverses for nutrient maxima. Concentrations and thresholds must use matching units, and the energy requirement E_{req} must use the same system as supply. Each solution requires its input matrix, solver status, constraint slack and active bounds.

$$C = \sum_{i=1}^5 c_i x_i; \quad D = \sum_{i=1}^5 d_i x_i \quad [1]$$

$$0.95DMI_{pred} \leq D \leq 1.05DMI_{pred} \quad [2]$$

$$\sum_{i=1}^5 (q_i - q_{min}) d_i x_i \geq 0 \quad [3]$$

$$\sum_{i=1}^5 ME_i d_i x_i \geq 1.05E_{req} \quad [4]$$

An anonymized analytical dataset and scripts are planned for open repository deposition. The package will include dated cow-month and feed data, prices, intake/calibration metadata, laboratory methods, validation identifiers, model predictions, residuals and formulation diagnostics. Commercial identifiers and confidential contract information require removal before release; restricted records require farm authorization.