

Estimation of Unsaturated Soil Hydrodynamic Parameters Using Inverse Problem Technique

J. M. V. Samani^{1*}, and P. Fathi²

ABSTRACT

Mathematical simulation of flow toward drains is an important and indispensable stage in drainage design and management. Many related models have been developed, but most of them simulate the saturated flow toward drains without a due consideration of the unsaturated zone. In this study, the two dimensional differential equation governing saturated and unsaturated flow in porous media is numerically solved and water table variations between drains predicted. By introducing and linking a proper optimization model to the numerical one, saturated and unsaturated soil hydrodynamic parameters were estimated within the inverse problem technique context. Data for calibration and verification were provided through a conduction of laboratory experimentation. Other laboratory data were also employed for the proposed model evaluation. The results indicated that in addition to a prediction of the water table variations between drains, the inverse problem model can be employed to estimate the unsaturated soil hydrodynamic parameters with a high degree of precision.

Keywords: Numerical model, Inverse problem technique, Saturated flow, Unsaturated flow.

INTRODUCTION

The unsaturated root-zone soil plays a significant role in moisture flow, salt transfer as well as in the growth of plants (Ryan, 2008). A drainage system extends the unsaturated zone through a drawdown of the water table. Due to the gravity part of the unsaturated zone, water flows toward drains as excess water.

Mathematical modeling of flow in saturated-unsaturated soil zone is an important tool through which soil moisture and water table variations between drains are predicted. Richard's equation is a partial differential equation describing soil water dynamics in modeling saturated-unsaturated flow in porous media (Clement *et al.*, 1994). Due to the nonlinearity of the equation, it

cannot be solved analytically and has to be solved by employing numerical methods (Simpson and Clement, 2003; Ramos *et al.*, 2006).

Simpson and Clement (2003) compared the application of finite difference and finite element methods as applied to porous media flows and demonstrated that the finite element methods are more precise than the finite difference ones. Whilliam *et al.* (1994) simulated saturated-unsaturated flow toward open drains using the finite difference method. They concluded that water table profile and seepage zone length along drain open sides are functions of capillary force in unsaturated zone. Fipps *et al.* (1986) solved partial differential equation of saturated-unsaturated flow numerically, using the finite element method. They showed that through a use of suitable boundary

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conditions, this method can precisely predict the water table depth near the drains. Other applications of finite difference and finite element methods for saturated-unsaturated flow in porous media can be found in literature (Clement *et al.*, 1994; Gureghian and Youngs, 1975; Nature *et al.*, 1975; Yeh and King, 1978; Fausey, 1975; Schwaerzel *et al.*, 2006; Vrugt *et al.*, 2001).

One of the serious problems encountered in drainage models is a determination of input parameters. This problem can restrict model applications for design and management purposes. Saturated and unsaturated soil hydraulic properties are the most important input parameters that vary spatially as well as temporally. Therefore, assigning values to these parameters is accompanied with error and uncertainty, making it necessary to find a new way to overcome the problem (Van Genuchten and Leij, 1992; Warrick and Myers, 1987; Moustafa, 2000; Vrugt *et al.*, 2001).

A new method of field measurement of soil parameters and their interpretation is an inverse problem technique. In this method, difficult direct measurements are achieved by measuring the easily available variables (Ritter *et al.*, 2003). This technique has many applications in engineering and such sciences as hydraulics and soil physics. Dane and Hruska (1983) used inverse problem technique to estimate soil moisture characteristic curve in homogeneous soils. Other researchers extended and improved the use of inverse problem technique in the field (Ritter *et al.*, 2003; Olyphant, 2003; Jhorar *et al.*, 2002; Abbaspour *et al.*, 1999; Stefan and Boris, 1999; Nutzman *et al.*, 1998; Simunek *et al.*, 1998; Kumar *et al.*, 1994; Bitterlich *et al.*, 2004).

In this research, the finite volume numerical method is employed to solve Richard's equation numerically for simulation of unsteady saturated-unsaturated flow toward drain conduits. Also, by employing the measured water table variations between drains, which is relatively easy as compared with direct measurements of unsaturated soil properties,

and the numerical model within the inverse problem technique context, average and effective values of saturated and unsaturated soil hydraulic properties can be estimated.

METHODOLOGY

Model Development

In the present research, the governing partial differential equation of saturated-unsaturated flow toward drains was solved numerically using the finite volume method and this model was then incorporated within the inverse problem technique. In numerical models, as compared to analytical ones, much more complicated geometry and boundary conditions can be analyzed.

Solution Domain

Figure 1 shows a vertical section, symbols and boundary conditions of a horizontal drainage system (point B is a drain). In this Figure, L is the distance between drains, H the water table height over an impermeable layer, d_e is the vertical distance between the drain and impermeable layer and H_0 an initial water table height over the impermeable layer.

Flow Equation

Richard's equation for a saturated-unsaturated 2-D flow in a heterogeneous and anisotropic soil is given as:

$$\frac{\partial}{\partial x} \left[K_x \frac{\partial H}{\partial x} \right] + \frac{\partial}{\partial y} \left[K_y \frac{\partial H}{\partial y} \right] = \frac{\partial \theta}{\partial t} \quad (1)$$

where H is the hydraulic head equal to $Z + h_p$; K_y and K_x are the vertical and horizontal unsaturated hydraulic conductivity for unsaturated condition respectively; θ is the volumetric soil moisture and Z the vertical distance to impermeable layer, being called gravitational potential; h_p is the

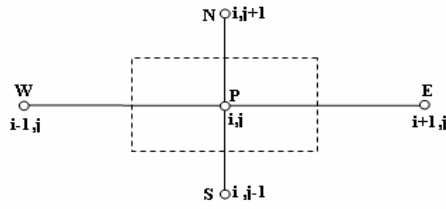


Figure 2. Schematic representation of 5 point grid system.

with five points along with Picard iteration scheme (Figure 2).

By integrating Equation (1), on the finite volume, the following equation is obtained:

$$H_{i,j}^{n+1,m+1} = \frac{a_i H_{i+1,j}^{n+1,m+1} + b_i H_{i-1,j}^{n+1,m+1} + c_i H_{i,j+1}^{n+1,m+1} + d_i H_{i,j-1}^{n+1,m+1} + e_i}{g_i} \quad (5)$$

where:

$$\begin{aligned} a_i &= \frac{k_{xe}^{n+1,m} \Delta y \Delta t}{\Delta x}, \quad b_i = \frac{k_{xw}^{n+1,m} \Delta y \Delta t}{\Delta x} \\ b_i &= \frac{k_{xw}^{n+1,m} \Delta y \Delta t}{\Delta x}, \quad d_i = \frac{k_{yn}^{n+1,m} \Delta x \Delta t}{\Delta y} \\ e_i &= C_{i,j}^{n+1,m} H_{i,j}^{n+1,m} \Delta x \Delta y - \left[\theta_{i,j}^{n+1,m} - \theta_{i,j}^n \right] \Delta x \Delta y \\ , \quad f_i &= C_{i,j}^{n+1,m} \Delta x \Delta y \\ g_i &= a_i + b_i + c_i + d_i + f_i \end{aligned} \quad (6)$$

In which m is the iteration number; n the time step; i and j are the node number for x and y directions respectively; $C_{i,j}^{n+1,m}$ is the moisture characteristic curve slope corresponding to the node (i,j) at $(n+1)_{th}$ time step and m_{th} iteration, $H_{i,j}^{n+1,m}$ is the hydraulic head at the node (i,j) at $(n+1)_{th}$ time step and m_{th} iteration, and $k_{yn}^{n+1,m}$ and $k_{xe}^{n+1,m}$ are the vertical and horizontal unsaturated hydraulic conductivities respectively in points N and E at $(n+1)_{th}$ time step and m_{th} iteration. Other notations are defined in a similar way.

Equation (4) is solved by Picard iteration method using the following algorithm:

1. Potential values are assumed for all nodes except for boundaries.

2. a_i to g_i coefficients are calculated.
3. Boundary conditions are applied.
4. Equation (4) is applied for all nodes with unknown potential as well as for its 4 neighborhood nodes, this procedure being applied for rows beginning from down left (Figure 2).

Through iterating stages (2) to (4) until convergence is achieved, all potentials would be determined.

Model Verification

The following steps are carried out for model verification:

- 1) When water table at drain location is assumed to be equal to initial water table, the water table predicted by the model is even all over the water table profile.
- 2) The proposed model results confirm the Fipps *et al.* (1986) finite element model's ones.

Optimization Procedure

Optimization procedure is necessary for estimating the different parameters involved, soil moisture characteristic curve and unsaturated hydraulic conductivity function parameters, within the inverse technique context. Most of the nonlinear optimization methods could have been used in this study. Box (1966) tested most of the optimization methods for functions with 2, 3, 5, 10, and 20 independent variables and stated that Powell's method performance is more appropriate than that of the other methods. Thus, this method was selected in the current study. The objective function identified in the computer code in this research is:

$$F = \sum_{i=1}^n (h_i^m - h_i^c)^2 \quad (7)$$

where h_i^m and h_i^c are the measured and calculated water table depths (hydraulic heads) above drains for point i respectively,

while n being the number of the measured points.

Model Calibration

In order to calibrate the model, experimental data are needed and therefore a laboratory model was set up. The laboratory model involved a drainage model made up of metal and Plexiglas equipped with piezometers for piezometric potential measurements. The physical model was filled with soil and left submerged for more than a month so that a stable soil structure could be established. In order to assess the necessary parameters, the drain outlet was closed and submergence conducted. Then the outlet was opened and the drawdown of water table profile with respect to time measured. Water height in piezometers (hydraulic head) was measured with time. Vertical and horizontal saturated soil hydraulic conductivities were determined using the falling head method, the obtained results being 9.65, and 8.86 cm day^{-1} respectively. For a measurement of soil moisture curve, pressure and ceramic plate membranes were employed. The data obtained by Pendy *et al.* were employed for the calibration process. In their research work, water table profiles and soil hydraulic conductivity of the laboratory model have been measured (Pendy *et al.*, 1992).

In this research, part of the water table data was used for calibration while the rest for verifying the obtained results. Calibration process was carried out by running the optimization computer program for water table heights, and hydraulic heads for various locations and related times, finally the parameter values being determined.

Sensitivity Analysis

By conducting sensitivity analysis, the effect of model input parameters on output, (water table profile) is determined. This process gives a good insight into the impact

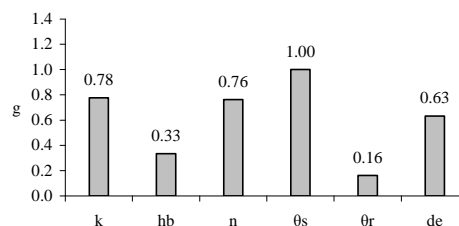


Figure 3. Values of γ for model input parameters.

of each parameter on the results and the applicability of the optimization process. Hill's method (Hill, 1998) was employed for calculating the sensitivity coefficient (γ) for each of the input parameters. Parameters with γ less than 0.1 cannot be estimated through the inverse technique. Figure 3 shows the sensitivity coefficient for different input parameters where θ_s and θ_r are saturated and residual soil moisture contents, respectively. As shown in this Figure, sensitivity coefficient for all parameters is greater than 0.1 and therefore it is concluded that the parameters can be calculated using the inverse technique.

Calibration as with Laboratory Data

As indicated previously, the purpose of calibration is to estimate optimal values for the involved parameters using laboratory data within the inverse technique program. The results are shown in Tables 1 and 2.

Calibration with Pendy *et al.* Data

Tables 3 and 4 show calibration results when Pendy *et al.*'s data is employed.

Verification

Two sets of data, laboratory and Pendy *et al.*'s, have been employed in the verification process. Using the soil hydrodynamic parameters obtained through the calibration

**Table 1.** Estimated and measured values of soil characteristic curve parameters.

Parameter	Value	
	Estimated	Measured
h_b (m)	0.0513	0.071
λ	0.0101	0.023
θ_s	0.398	0.418
θ_r	0.097	0.0521

Table 2. Estimated and measured values of horizontal and vertical unsaturated hydraulic conductivity parameters.

Parameter	Value	
	Estimated	Measured
$K_{sx} (\frac{m}{day})$	0.101	0.0965
$K_{sy} (\frac{m}{day})$	0.0886	0.0804
h_b (m)	0.0513	0.071
η	2.03	2.069

procedure water table profiles, not employed in the calibration procedure, were predicted for the two verification cases.

Verification Using Laboratory Data

Using the soil hydrodynamic parameters obtained through the calibration procedure, water table profiles were predicted for 90 and 150 minutes after drainage started. The water table profile, hydraulic heads, after a lapse of 90 minutes is shown in Figure 4. Also water table variations with time, calculated through measured and estimated parameters, are shown in Figure 5.

Verification Using Pendy *et al.* Data

For the verification, calculated and measured water table profiles after a 3 day lapse from drainage beginning, using Pendy *et al.* data, are shown in Figure 6. Also water table variations midway between drains with

Table 3. Values of constant parameters of soil moisture characteristic curve, using Pendy *et al.* (1992) data.

Parameter	Value	
	Estimated	Measured
h_b (m)	0.308	-
λ	0.044	-
θ_s	0.4	0.44
θ_r	0.051	-

Table 4. Values of constant parameters of horizontal and vertical unsaturated hydraulic conductivity function, using Pendy *et al.* (1992) data.

Parameter	Value	
	Estimated	Measured
$K_{sx} (\frac{m}{day})$	0.035	0.0375
$K_{sy} (\frac{m}{day})$	0.04	0.04
h_b (m)	0.308	-
η	2.132	-

respect to time are shown in Figure 7. As the measured soil moisture curve values were not available in Pendy *et al.*'s research, therefore prediction of water table profiles using measured parameters did not become possible.

To investigate and quantify the unsaturated vertical flow to drains using the estimated parameters, pressure potential magnitudes at different distances from drains (0.5, 5 and 10 days following the beginning of drainage) beginning are shown in Figure 8. In this Figure, zero pressure potential indicates water table. Figure 9 shows vertical and horizontal water velocity components as predicted by the model.

To evaluate the model statistically, such indices as RMSE (Root Mean Square Error), ME (Modeling Efficiency), EF (Efficiency) and MAPE (Mean Absolute Percentage) were employed. These indices can be expressed as follows (Homaei *et al.*, 2002):

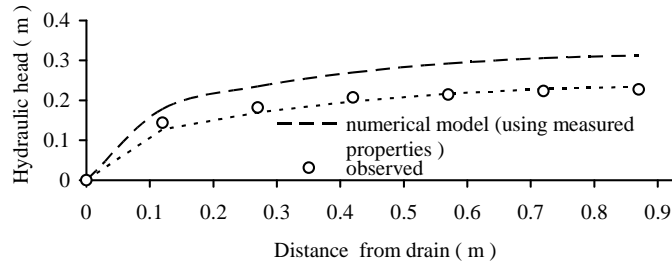


Figure 4. Observed and computed profile of water table, 90 minutes after initiation of drainage.

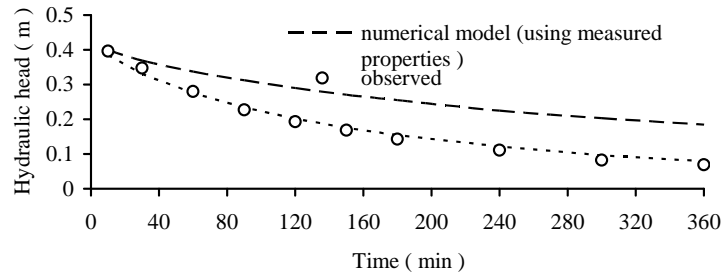


Figure 5. Observed and computed water table midway between drains.

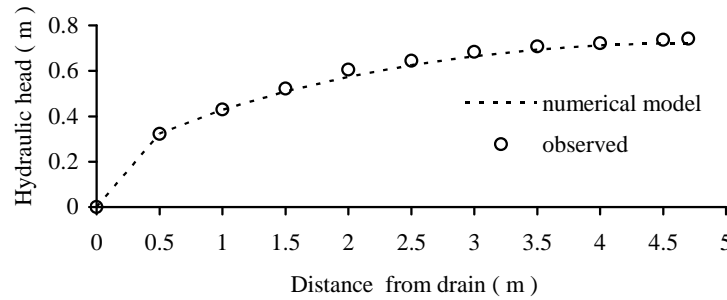


Figure 6. Observed and computed based profile of water table, 3 days after initiation of drainage.

$$RMSE = \left[\frac{\sum_{i=1}^n (h_i^c - h_i^m)^2}{n} \right]^{1/2} \quad (8)$$

$$ME = \max \left| h_i^c - h_i^m \right|_{i=1}^n \quad (9)$$

$$EF = \frac{\sum_{i=1}^n (h_i^m - \bar{h})^2 - \sum_{i=1}^n (h_i^c - \bar{h})^2}{\sum_{i=1}^n (h_i^m - \bar{h})^2} \quad (10)$$

$$MAPE = \frac{\sum_{i=1}^n \left| \frac{h_i^c - h_i^m}{h_i^m} \right|}{n} \quad (11)$$

where h_i^c and h_i^m are the calculated and measured hydraulic heads, respectively, \bar{h} is the average hydraulic head and n the number of measured points. The indices related to the model are shown in Tables 5 and 6.



RESULTS AND DISCUSSION

Figures 4 to 7 and Tables 5 to 6 imply good agreement between measured and predicted water table hydraulic heads. The profiles predicted by employing the

estimated unsaturated soil hydrodynamic characteristics are much more indicative than those predicted while using the measured ones, indicating the importance of measurement error effects on the results.

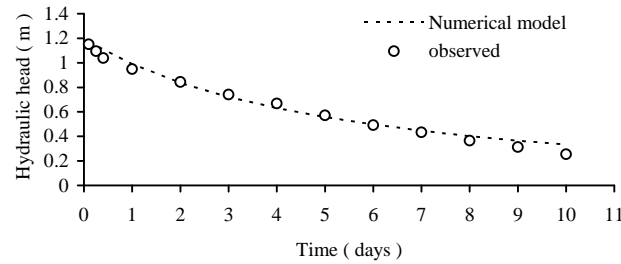


Figure 7. Observed and computed based water table, midway between drains.

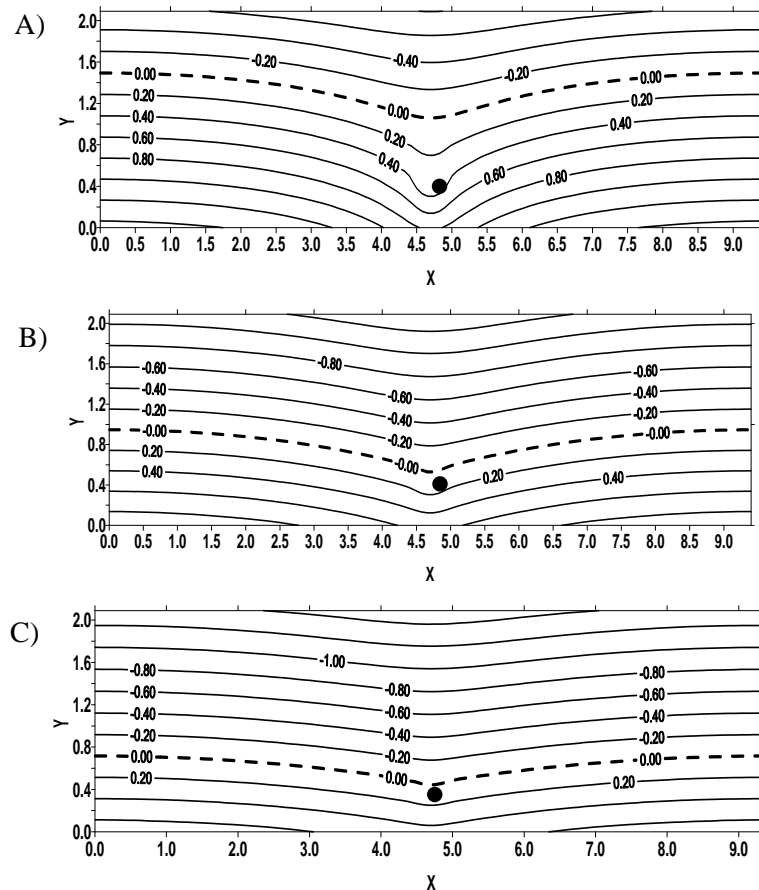


Figure 8. Isobar of soil at: (A) 0.5, (B) 5 and (C) 10 days after initiation of drainage.

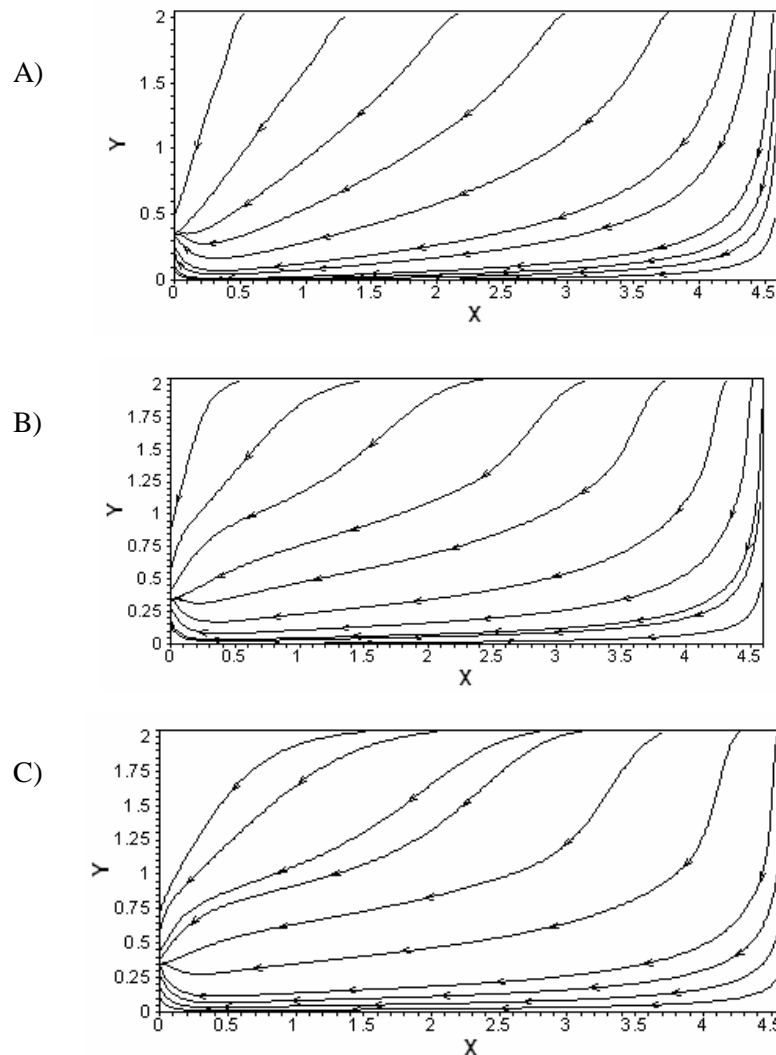


Figure 9. Pathlines towards drains at: (A) 0.5, (B) 5 and (C) 10 days after initiation of drainage.

The present research indicates the advantage of the proposed method in estimating the unsaturated soil hydrodynamic characteristics using water table profile variations, it being concluded that flow condition is playing an important role in an estimation of the parameters.

Figures 8 and 9 indicate that unsaturated zone plays an important role in flow close to drains and its effect becomes more significant with a lapse of time and as the unsaturated zone becomes extended. Therefore, the proposed 2-D model of

saturated-unsaturated drainage model can serve as a proper tool to fulfill the objectives of this research.

Finally, the results indicate that linking the numerical model solution to the inverse problem technique introduces an effective tool capable of estimating unsaturated soil hydrodynamic characteristics, necessary in both design and management of drainage systems.

**Table 5.** Values of statistical indices (Calculated using measured data).

Statistical indices	Numerical model (Using estimated properties)	Numerical model (Using measured properties)
RMSE (m)	0.0099	0.0847
ME (m)	0.0271	0.121
EF (%)	97.65	73.1
MAPE (%)	6.8	75.81

Table 6. Values of statistical indices (Calculated using Pendy *et al.* (1992) data) .

Indices	Value
RMSE (m)	0.0275
ME (m)	0.075
EF (%)	97.71
MAPE (%)	5.7

REFERENCES

1. Abbaspour, K., Sonnleitner, M. A. and Schulin, R. 1999. Uncertainty in Estimation of Soil Hydraulic Properties by Inverse Modeling: Example Lysimeter Experiments. *Soil Sci. Soc. Am. J.*, **63**: 501-509.
2. Bitterlich S., Durner W., Iden S. C. and Knabner P. 2004. Inverse Estimation of the Unsaturated Soil Hydraulic Properties from Column Outflow Experiments Using Free-form Parameterization. *Vadose Zone J.*, **3**(3): 971-981.
3. Box, M. J. 1966. A Comparison of Several Current Optimization Methods and Use of Transformation in Constrained Problems. *The Computer J.*, **9**: 67-77.
4. Brooks, R. H., and Corey, A. T. 1964. Hydraulic Properties of Porous Media. Hydrol. Pap. 3, Colo. State Univ., Fort Collins.
5. Clement, T. P., Wise, W. R. and Molz, F. J. 1994. A Physically-based, Two-dimensional, Finite Difference Algorithm for Modeling Variably Saturated Flow. *J. Hydrol.*, **161**: 71-90.
6. Dane, J. H. and Hruska, S. 1983. *In-situ* Determination of Soil Hydraulic Properties During Drainage. *Soil Sci. Soc. Am. J.*, **47**: 619 – 624.
7. Fausey, N. R. 1975. Numerical Model Analysis and Field Study of Shallow Subsurface Drainage. Ph. D. Dissertation, Ohio State Univ., Columbus , OH, 144 P.
8. Fipps, G., Skaggs, R. W. and Nieber, J. L. 1986. Drains as a Boundary Condition in Finite Elements. *Water Resour. Res.*, **22**(11): 1613-1621.
9. Gureghian, A. B. and Youngs, E. G. 1975. The Calculation of Steady-state Water Table Heights in Drained Soils by Means of the Finite Element Method. *J. Hydrol.*, **27**: 15-32.
10. Hill, M. C. 1998. Methods and Guidelines for Effective Model Calibration. U.S. Geological Survey Water-resources Investigations Rep. 98-4005, Available at: <http://water.usgs.gov/software/ucode.html> [Modified 4. 2002; Verified 144-Feb. 2003. U.S. Geol. Survey, Reston, VA.].
11. Homaei, M., Dirksen, C. and Feddes, R. A. 2002. Simulation of Root Water Uptake. I. Non-uniform Transient Salinity Using Different Macroscopic Reduction Functions. *Agric. Water Manage.*, **57**: 89-109
12. Jhorar, R. K., Bastiaanssen, W. G. M., Feddes, R. A. and Van Dam, J. C. 2002. Inversely Estimating Soil Hydraulic Functions Using Evapotranspiration Fluxes. *J. Hydrol.*, **258**: 198-213.
13. Kumar, S., Gupta, S. K. and Ram, S. 1994. Inverse Techniques for Estimating Transmissivity and Drainable Pore Space Utilizing Data from Subsurface Drainage Experiments. *Agric. Water Manage.*, **26**: 41-58.
14. Moustafa, M. M. 2000. A Geostatistical Approach to Optimize the Determination of Saturated Hydraulic Conductivity for Large-

15. Nature, F. S., King, L. G. and Jeppson, R. W. 1975. Unsaturated-saturated Flow Through Heterogeneous Sloping Lands. Rep. PRWG59c-9, Utah Water Res. Lab., Logan, 45 P.
16. Nuttmann, M., Thiele, S. M. and Joswig, K. 1998. Inverse Modelling Techniques for Determining Hydraulic Properties of Porous Media by Transient Outflows Methods. *Advances in Water Resources*, **22**(3): 273 – 284.
17. Olyphant, G. A. 2003. Temporal and Spatial (Down Profile) Variability of Unsaturated Soil Hydraulic Properties Determined From a Combination of Repeated Field Experiments and Inverse Modeling. *J. Hydrol.*, **281**(1-2): 37-43.
18. Pendy, R. S., Bhattacharya, A. K., Singh, O. P. and Gupta, S. K. 1992. Drawdown Solution with Variable Drainage Porosity. *J. of Irrigation and Drainage Eng.*, **118**(3): 382-396.
19. Ritter, A., Hupet, F., Carpena, R. M., Lambot, S. and Van Clooster, M. 2003. Using Inverse Methods for Estimating Soil Hydraulic Properties from Field Data as an Alternative to Direct Methods. *Agric. Water Manage.*, **59**: 77-96.
20. Ryan, J. 2008. Crop Nutrients for Sustainable Agricultural Production in the Drought-Stressed Mediterranean Region. *J. Agric. Sci. Technol.* **10**:295-306
21. Schwaerzel K., Simunek J., Stoffregen H., Wessolek G. and Van Genuchten M. T. 2006. Estimation of Unsaturated Soil Hydraulic Conductivity of Peat Soils; laboratory Versus Field Data. *Vadose Zone J.*, **5**(2):628-640.
22. Simpson, M. J. and Clement, T. P. 2003. Comparison of Finite Difference and Finite element Solutions to the Variably-Saturated Flow Equation. *J. Hydrol.*, **270**: 49-64
23. Simunek, J. and Hopmans, J. W. 1998. Parameter Estimation of Unsaturated Soil Hydraulic Properties from Transient Flow Process. *Soil Till. Res.*, **47**(1- 2): 27–36.
24. Simunek, J., Jaramillo, R. A., Schaap, M. G., Vandervear, J. P. and Van Genuchten, M. Th. 1998. Using an Inverse Method to Estimate the Hydraulic Properties of Crusted Soils from Tension-Disk Infiltrometer Data. *Geoderma*, **86**(1-2): 61 – 81.
25. Stefan, F. and Boris, F. 1999. Inverse Modelling of a Radial Multistep Outflow Experiment for Determining Unsaturated Hydraulic Properties. *Advances in Water Resources*, **22** (5): 431-444
26. Tiago, B., Ramos, M. C., Goncalves, J. C. M., Van Genuchten, M. T. and Pires, F. P. 2006. Estimation of Soil Hydraulic Properties from Numerical Inversion of Tension Disk Infiltrometer Data. *Vadose Zone J.*, **5**(2):684-696.
27. Van Genuchten, M. Th. and Leij, F. J. 1992. On Estimating the Hydraulic Properties of Unsaturated Soils. Proc. of the International Workshop on Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils, The Netherlands.
28. Vrugt, J. A., Bouten, W., Gupta H. V., and Sorooshian, S. 2001. Toward Improved Identifiability of Hydrologic Model Parameters: The Information Content of Experimental Data. *Water Resour. Res.*, **38**(12): 48-53.
29. Vrugt, J. A., Hopmans, J. W. and Simunek J. 2001. Calibration of a Two-dimensional Root Water Uptake Model. *Soil Sci. Soc. Am. J.*, **64**(4): 1027-1037.
30. Warrick, A. W. and Myers, D. E. 1987. Calculations of Error Variances with Standardized Variograms. *Soil Sci. Soc. Am. J.*, **51**: 265-268.
31. Wise, W. R., Clement, T. P. and Molz, F. J. 1994. Variably-Saturated Modeling of Transient Drainage: Sensivity to soil parameters. *J. Hydrol.*, **161**: 91-108
32. Yeh, H. L. and King, L. G. 1978. Transient Subsurface Drainage on Sloping Irrigated Land. Paper Presented at the Summer Meeting, *Am. Soc. Agric. Eng.*, Logan, Utah.



تخمین پارامترهای هیدرودینامیکی غیر اشباع خاک با استفاده از مسأله معکوس

ج. م. و. سامانی و پ. فتحی

چکیده

شبیه‌سازی ریاضی جریان به طرف زهکش‌ها مهمترین قدم در طراحی و مدیریت سیستمهای زهکشی به شمار می‌آید. اگر چه مدل‌های زهکشی مختلفی توسط محققان بسط و توسعه داده شده است. اما اکثر این مدل‌ها تنها قادر به مدل‌سازی جریان اشباع به طرف زهکش‌ها می‌باشند، در حالی که در سیستمهای زهکشی جریان به طور توأم در دو ناحیه اشباع و غیر اشباع خاک اتفاق می‌افتد. در این تحقیق با منفصل سازی معادله دیفرانسیل حاکم بر جریان اشباع- غیر اشباع به طرف زهکش و اعمال شرایط مرزی مناسب، مدلی عددی جهت پیش‌بینی نوسانات سطح ایستابی در اطراف زهکش بسط داده شد. با انتخاب الگوریتم بهینه‌سازی مناسب و اتصال آن به مدل عددی، مدل معکوس مناسب جهت برآورد مقادیر میانگین توابع آبی غیر اشباع خاک طراحی گردید. با طراحی و ساخت مدل فیزیکی زهکشی در آزمایشگاه، داده‌های مورد نیاز جهت واسنجی و آزمون مدل پیشنهادی اندازه‌گیری شد. از داده‌های آزمایشگاهی محققان دیگر نیز برای ارزیابی دقت پیش‌بینی مدل پیشنهادی استفاده شد. نتایج حاصل از تحقیق نشان داد، با استفاده از مدل معکوس پیشنهادی مقادیر متوسط ویژگیهای هیدرودینامیک غیر اشباع خاک را با دقت بالایی می‌توان برآورد کرد. با استفاده از مقادیر برآوردی از مدل می‌توان نوسانات سطح ایستابی در اطراف زهکش را با دقت بیشتری پیش‌بینی کرد.