Reservoir Routing through Successive Rockfill Detention Dams

J. M. V. Samani1* and M. Heydari1

ABSTRACT

Rock has been advantageously employed in hydraulic structures such as rockfill dams, gabion weirs and drain works. One rockfill dam application can be flood control in watershed management. The objectives of building rockfill detention dams are flow storage for a specific period and lowering of the outflow hydrograph. As this type of dam consists of coarse particles, seepage flow will deviate from Darcy’s law and mostly be turbulent. Under the practical conditions of watershed management, it might be necessary to build successive rockfill dams, where a final outflow hydrograph with lower peak flows and longer duration is needed. Due to their reciprocal effects, the hydraulics of successive rockfill detention dams are complicated. This paper describes a routing flow model through successive rockfill dams considering the storage among them and their effects on each other. In the developed model, the velocity has been introduced to the 1-D continuity equation as an exponential relationship between Reynolds number (Re) and the Darcy-Weisbach friction factor (f). By introducing the inflow hydrograph and rockfill characteristics as input data to the model, the outflow hydrograph can be determined through the storage routing method. The results of the developed model show good agreement with the experimental data collected for this investigation. The results show that the degree of peak reduction of the routed hydrograph depends on the number of successive rockfill dams, the distance between them, the average size of the rockfill material, and the dam dimensions.

Keywords: Non-Darcy flow, Reservoir routing, Rockfill dams.

INTRODUCTION

Many hydraulic structures have been constructed to utilize the water from rivers, but almost all of them have been made of concrete or steel in order to utilize the water to the utmost limit. These kinds of structures interrupt the natural flow of the river and reduce the auto-purification effect of the river. Therefore such structures have had a negative influence on the habitat environment of the river. With such circumstances as a background, nature-friendly river designing has been attracting attention in recent years. Rock can be used to build gabions, spillways, and groins (Stephenson, 1979) and a rockfill dam made of rocks is expected to be a suitable structure for the river.

Rockfill dam application is an economic and useful method for flood management purposes when suitable rock is available. Rockfill dams can be designed satisfactorily when the hydraulics of flow through the rockfill dam are known. This type of dam consists of coarse particles, and so the flow will deviate from Darcy’s law and be mostly turbulent. This means that the relationship between the flow velocity, $V$, and the hydraulic gradient, $i$, is nonlinear. Various researchers have proposed the following

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nonlinear relationships (Herrera and Felton, 1991):
\[ i = A V^b \]  
\[ i = A' V + B' V^2 \]  
where \( A, A', B, \) and \( B' \) are coefficients depending on the rock and fluid characteristics. Equations (1) and (2) were proposed by Prony in 1804 and Forcheimer in 1901, respectively (Li et al., 1998). Other researchers suggested relationships between Reynold's number (Re) and the Darcy-Weisbach friction factor (f) in the following forms (Herrera and Felton, 1991):
\[ f = a b \]  
\[ f = a' + b' \]  
where \( a, a', b, \) and \( b' \) are also coefficients which depend on the rock and fluid characteristics. Reynold's number is defined as:
\[ \text{Re} = \frac{V(d - \sigma)}{\nu} \] where \( d \) is the average size of rock particles, \( \sigma \) is the standard deviation of the rock size distribution and \( \nu \) is kinematic viscosity. If the Reynold's number is written in terms of \( V \), the Darcy-Weisbach friction factor can be expressed in the form of Equations (1) and (2), respectively. Various researchers, such as Ergun (1989), Wilkins (1956), Ward (1964), Leps (1973), McCorquodale et al. (1978), Stephenson (1979), Herrera and Felton (1991), Hansen et al. (1995), Bingium et al. (1998), and Kataraman and Ramp (1998) proposed the above equations in their research. Findikakis and Tu (1985) introduced an equation to simulate flood routing by computing the flow profile through a rock dump using the continuity equation. A review of the different relationships proposed by various researchers was under taken by Samani et al. (2003). They proposed 1-D and 2-D models for flow through rockfill dams (Samani et al., 2003). In their 1-D model, the following relationship has been used:
\[ f = 54.0 \text{Re}^{-0.077} \]  
In the present paper, a model is proposed to solve the problem of routing flow through successive rockfill dams considering the storage among rockfill dams reservoirs and their reciprocal effects on each other. The model is based on the 1-D continuity equation, which employs Eq.5 within the storage routing method.

Model Development and Solution

Flow Rating Equation

To develop the flow rating equation, it was shown (Samani et al., 2003) that:
\[ V = \alpha \text{Re}^{\frac{1}{3+b}} \]  
where
\[ \alpha = \left( \frac{2 g \nu}{a(d - \sigma)^{b-1}} \right)^{\frac{1}{3+b}} \]  
In Eq.7, \( a \) and \( b \) are Eq.3 coefficients, \( \nu \) is kinematic viscosity, \( d \) is average size of rock particles, \( g \) is acceleration due to gravity and \( \sigma \) is the standard deviation of the rock size distribution.

Combining Eq.6 with the continuity equation and defining \( i \) as \( \left(-\frac{dh}{dx}\right) \), yields:
\[ Q = \alpha \left(-\frac{dh}{dx}\right)^{\frac{1}{3+b}} h w \]  
where \( Q \) is outflow rate through dam, \( h \) is water depth inside the rockfill dam, \( w \) is the width of flow cross section, \( x \) is the longitudinal coordinate in the flow direction. Integrating Eq.8 between the limits \( H_{up} \) to \( H_{down} \) for \( h \) and zero to \( D \) for \( x \) gives the following:
\[ Q = \left(\frac{1}{D}\right)^{\frac{1}{3+b}} \frac{\alpha w}{(3+b)^{\frac{1}{3+b}}} \left(H_{up}^{b+3} - H_{down}^{b+3}\right)^{\frac{1}{3+b}} \]  
where, according to Figure 1, \( D \) is defined according to Sharma (1991) as:
\[ D = L - 0.7 S_l \]  
\[ S_l = H_{up} \cot \beta \]
In Eq.10, $\beta$ is the angle of the upstream and downstream face of the dam with the horizontal direction, $H_{up}$ and $H_{down}$ refer to dam upstream and downstream water depths, respectively, and $L$ is dam length according to flow direction. Eq.9 is the flow rating equation for 1-D flow through rockfill dams (Samani et al., 2003).

Reservoir Routing

Figure 2. shows a number of rockfill dams successively located along the same path. During a flood, flow volume is stored among the successive reservoirs and, accordingly, the outflow hydrograph is lowered significantly. Due to the successive storages, the routed outflow hydrograph will experience a reduced peak and the time required to reach a safe peak will increase. The following discussion shows the set of equations needed for simulating the flow through successive dams and reservoirs.

In this investigation, it is assumed that the flow in the reservoir has no significant velocity.

The basic equation for flow routing is:

$$I - O = \frac{dS}{dt}$$

where $I$ is reservoir inflow rate, $O$ is reservoir outflow rate, and $\frac{dS}{dt}$ is storage variation with respect to time. The finite difference form of Eq.11 for the first reservoir is:

$$I_i^{(1)} + I_i^{(1)} - O_i^{(1)} + O_i^{(1)} = \frac{S_i^{(1)} - S_i^{(1)}}{\Delta t}$$

where $i$ and $i+1$ indicate successive time steps for a time increment equal to $\Delta t$ and according to Figure 2. The superscript refers to the reservoir number. For the second reservoir, the routing equation becomes:

$$I_i^{(2)} + I_i^{(2)} - O_i^{(2)} + O_i^{(2)} = \frac{S_i^{(2)} - S_i^{(2)}}{\Delta t}$$

Figure 2. Routing through successive detention rockfill dams.
where \( I^{(2)}_i = O^{(1)}_i \) and \( I^{(2)}_{i+1} = O^{(1)}_{i+1} \).

Substituting the outflow from reservoir No.1 for the inflow to reservoir No.2 in Eq.13 yields:
\[
\frac{O^{(1)}_{i+1} + O^{(1)}_i}{2} - \frac{O^{(2)}_{i+1} + O^{(2)}_i}{2} = \frac{S^{(1)}_{i+1} - S^{(1)}_i}{\Delta t}
\]
Eq.11 can be written for the third reservoir as:
\[
\frac{I^{(3)}_i + I^{(3)}_{i+1}}{2} - \frac{O^{(3)}_i + O^{(3)}_{i+1}}{2} = \frac{S^{(2)}_i - S^{(2)}_{i+1}}{\Delta t}
\] (15)
and as \( I^{(2)}_i = O^{(1)}_i \) and \( I^{(2)}_{i+1} = O^{(1)}_{i+1} \), Eq.15 becomes:
\[
\frac{O^{(2)}_i + O^{(2)}_{i+1}}{2} - \frac{O^{(3)}_i + O^{(3)}_{i+1}}{2} = \frac{S^{(1)}_i - S^{(1)}_{i+1}}{\Delta t}
\] (16)

In the same manner it is possible to extend the above concepts to \( P \) successive dams as follows:
\[
\frac{O^{(p)}_{i+1} + O^{(p)}_i}{2} - \frac{O^{(p+1)}_{i+1} + O^{(p+1)}_i}{2} = \frac{S^{(p)}_i - S^{(p)}_{i+1}}{\Delta t}
\] (17)

Last equation can be written as:
\[
O^{(p)}_i = Q(H^{(p+1)}_i) \Rightarrow O^{(p)}_i = Q(H^{(p+1)}_i)
\] (18)
where \( Q \) is flow rate and \( Q(H^{(p+1)}_i) \) is downstream flow rating relationship.

The general form of flow rating equation, Eq.9, for calculating \( O \) of each of the rockfill dams is:
\[
O = \left( \frac{1}{D} \right) \left( \frac{a}{(b+3)x^2} \right) \left( H^{(p+1)}_i - H^{(p)}_i \right)^2
\] (19)
where \( H^{(p+1)}_i \) and \( H^{(p)}_i \) refer rockfill upstream and downstream water depths, respectively. Therefore if \( P = 3 \), four equations (Eq.12, Eq.14, Eq.16 and Eq.18) will be available to be solved for four unknowns \( H^{(1)}_i, H^{(2)}_i, H^{(3)}_i \) and \( H^{(4)}_i \), and for \( P \) dams, \( P+1 \) unknowns need to be calculated.

In four equations (Eq.12, Eq.14, Eq.16 and Eq.18), \( S^{(1)}_{i+1}, O^{(1)}_{i+1}, S^{(2)}_{i+1}, O^{(2)}_{i+1}, S^{(3)}_i \) and \( O^{(3)}_{i+1} \) are the unknowns which give us:
\[
O^{(1)}_{i+1} = f(H^{(1)}_i, H^{(2)}_i)
\] (20)
\[
S^{(1)}_{i+1} = f(H^{(1)}_i)
\] (21)
\[
O^{(2)}_{i+1} = f(H^{(2)}_i, H^{(3)}_i)
\] (22)
\[
S^{(2)}_i = f(H^{(2)}_i)
\] (23)
\[
O^{(3)}_i = f(H^{(3)}_i, H^{(4)}_i)
\] (24)
\[
S^{(3)}_i = f(H^{(3)}_{i+1})
\] (25)
\[
O^{(3)}_{i+1} = f(H^{(4)}_{i+1})
\] (26)

Therefore, the actual unknowns are \( H^{(1)}_i, H^{(2)}_i, H^{(3)}_i \) and \( H^{(4)}_i \).

**Solution**

By using of Eq.19 for each of the dams and Eq.18 as the downstream flow rating relationship for \( P \) dams, the generalized equations of the problem are summarized below as:
\[
\frac{O^{(1)}_{i+1} + O^{(1)}_i}{2} - \frac{O^{(2)}_{i+1} + O^{(2)}_i}{2} = \frac{S^{(1)}_{i+1} - S^{(1)}_i}{\Delta t}
\] (12)
\[
\frac{O^{(2)}_i + O^{(2)}_{i+1}}{2} - \frac{O^{(3)}_i + O^{(3)}_{i+1}}{2} = \frac{S^{(2)}_i - S^{(2)}_{i+1}}{\Delta t}
\] (14)
\[
\frac{O^{(3)}_i + O^{(3)}_{i+1}}{2} - \frac{O^{(4)}_i + O^{(4)}_{i+1}}{2} = \frac{S^{(3)}_i - S^{(3)}_{i+1}}{\Delta t}
\] (16)

\[
O^{(p)}_{i+1} = Q(H^{(p+1)}_i)
\] (18)
where the unknowns are \( H^{(1)}_i, H^{(2)}_i, H^{(3)}_i \) and \( H^{(4)}_i \). To solve this model for the \( P+1 \) unknown, the following information is needed:

- Equation 19 for each dam,
- Inflow hydrograph for the first reservoir,
- Volume-elevation relationship for each reservoir,
- Rockfill characteristics for each dam, and,
- Downstream flow rating relationship.

**Numerical Procedure**

To solve the previously listed set of equations a computer program was developed on the basis of the Gauss-Seidel iterative method algorithm and it can be defined as follows:

1. Set initial condition for \( H^{(1)}_i, H^{(2)}_i, \)
Reservoir Routing through Successive Rockfill...  

\[ H_i^{(3)}, ..., H_i^{(P+1)} \] In the inflow hydrograph, the initial condition \((t = t_1 \text{ and } Q = Q_1)\) is known. Therefore water depths for \((i = 1)\) are calculable by using Eq.27.

\[ O_l^{(1)} = O_l^{(2)} = \ldots = O_l^{(P)} = Q(H_i^{(P+1)}) = Q_i \]

2. Read inflows, \(I_i^{(1)}, I_i^{(1)}\) for the selected \(\Delta t\) from the inflow hydrograph.

3. For running the iterative method, \(H_i^{(1)}, H_i^{(2)}, H_i^{(3)}, ..., H_i^{(P+1)}\) are assumed. The best assumption for \(H_i^{(1)}, H_i^{(2)}, H_i^{(3)}, ..., H_i^{(P+1)}\) is \(H_i^{(1)}, H_i^{(2)}, H_i^{(3)}, ..., H_i^{(P+1)}\), respectively.

4. Calculate \(S_i^{(1)}, S_i^{(2)}, ..., S_i^{(P)}\) using the information from step 1 and the volume-elevation relationship

5. Calculate \(O_i^{(1)}, O_i^{(2)}, ..., O_i^{(P)}\) using the information from steps 1 and 2 and Eq.19.

6. Calculate \(S_i^{(1)}, S_i^{(2)}, ..., S_i^{(P)}\) using the information from step 3 and the volume-elevation relationship for each reservoir,

7. Calculate \(O_i^{(1)}, O_i^{(2)}, ..., O_i^{(P)}\) using the information from step 3 and Eq.19.

8. Calculate \(H_i^{(P+1)}\) using Eq.18.

9. Solve for \(O_i^{(3)}, O_i^{(2)}, ..., O_i^{(P)}\) using Equations 12, 14, 16, ..., 17, respectively.

10. Compare the results of steps 9 and 7 and repeat steps 3 to 9 until convergence occurs.

The numerical procedure can be conducted provided that \(a\) and \(b\) in Eq.3 are known. Due to the sensitivity of \(H_i\) to the range of average size of rock material particles less than 20 mm (Samani et al., 2003), a calibration for \(a\) and \(b\) using the experimental data collected in this investigation has been conducted.

**Experimental Data**

Experimental data were needed for the calibration and validation of the mathematical model. The experiments were conducted in a laboratory channel 10.0 m long, 0.3 m wide, and 0.45 m high. Different cases were investigated as follows by changing the characteristic rockfill particle diameter, rockfill dam length, distance between rockfill dams, and number of rockfill dams with \(\beta = 90\) degrees:

- Two rockfill dams with average particle size of 14.5 mm, length of 0.4 m and distance between dams of 0.4 m;
- Two rockfill dams with average particle size of 14.5 mm, length of 0.4 m and distance between dams of 0.75 m;
- Two rockfill dams with average particle size of 21.0 mm, length of 0.5 m and distance between dams of 0.5 m;
- Two rockfill dams with average particle size of 21.0 mm, length of 0.5 m and distance between dams of 1.0 m;
- Three rockfill dams with average particle size of 14.5 mm, length of 0.4 m and distance between dams of 0.75 m;
- Three rockfill dams with average particle size of 21.0 mm, length of 0.5 m and distance between dams of 1.0 m.

The inflow hydrograph to the first rockfill reservoir was measured by a triangular weir installed at the beginning of the reservoir. This hydrograph was established with a programmable electrical valve. The outflow hydrograph was measured using a downstream channel rating curve. For measuring water level variation along the channel, each reservoir and downstream channel, a number of sensitive digital point gauges were installed. Each point gauge was equipped with memory storage to record water levels during the routing procedure. To get average particle size about 20.0 mm, particle rocks were sifted (sieved) through two sieves with openings of 21.0 mm and 19.0 mm, respectively.

To hold rockfill dams in their positions each rockfill was equipped by a thin galvanized basket. Finally, the output of the experiment was six routed outflow hydrographs which were used in calibrating and validating the mathematical model.
Model Verification, Calibration and Evaluation

For verifying the model, the following steps were taken by an assumed inflow hydrograph and assumed characteristics of successive rockfill dams:

a) Checking the computer program by imposing equal water elevations for the upstream and downstream levels of each of the dams. This condition introduced a zero flow rate through the rockfill dam and, if the water levels in all reservoirs are equal, the outflow of the successive rockfill dams will be zero, too.

b) Applying a constant inflow rate shows a stable water level for each reservoir indicating steady state flow conditions.

c) The model flood routing results show that the volume of the first reservoir inflow hydrograph is equal to the outflow hydrograph volume of each of the rockfill dams.

d) Applying the model for a very short length between successive rockfill dams considering the small separation between dams will introduce outflow hydrographs very close to the inflow hydrographs in terms of magnitudes and duration.

The conclusion from the tests was positive indicating the validity of the mathematical model.

Calibration for Eq. 3 was conducted using 50% of the collected data and a nonlinear optimization program. The results are \( a = 54.0 \) and \( b = -0.077 \).

The conclusions from the tests were positive indicating the validity of the mathematical model.

For the validation of the model, the complete data were used as below:

The model has been applied for the six routing cases so that the mathematical model can be evaluated. Figures 3 to 8 show the routed measured and calculated hydrographs. The agreement between the measured and calculated hydrographs is quite reasonable. The \( R^2 \) (regression coefficient) of all the figures is more than 91%, proving the validity of the mathematical model.

Sensitivity

At this stage, where the mathematical model has been validated, the sensitivity associated with the important model parameters can be evaluated. Sensitivity was tested by assuming an inflow hydrograph and successive rockfill dams. This investigation shows that each parameter has a different effect on \( \Delta Q\% \) and \( \Delta T\% \), where \( \Delta Q\% \) is the percentage of the difference between the peaks of the last outflow hydrograph and the first inflow hydrograph relative to the first inflow hydrograph peak and \( \Delta T\% \) is the re-

![Figure 3](image-url)

**Figure 3.** Two rockfill dams with a average particle size of 14.5 mm, length of 0.4 m and distance of 0.4 m (\( R^2 = 97\% \)).
lated percentage difference in the time to peak, respectively. Table 1 demonstrates the results of the sensitivity of the parameters. Each of the investigated parameters has a

Figure 4. Two rockfill dams with an average particle size of 14.5 mm length of 0.4 m and distance between dams of 0.75 m ($R^2=97\%$).

Figure 5. Two rockfill dams with an average particle size of 21.0 mm length of 0.5 m and distance between dams of 0.5 m ($R^2=96\%$).

Figure 6. Two rockfill dams with an average particle size of 21.0 mm length of 0.5 m and distance between dams of 1.0 m ($R^2=96\%$).
Among the investigated parameters, $d_{50}$ is the most important parameter compared to the others. The larger the $d_{50}$, the bigger the outflow hydrograph peak and the shorter the related period will be. Longer $L$ implies more head losses, a lower outflow peak and a longer relative time difference between peaks. For the constant $L$, $\beta$ would have the same effect as $L$ meaning that larger $\beta$ results in more head losses and outflow hydrograph dampening.

**CONCLUSION**

In the present study, a model has been presented to solve the problem of flow routing through successive rockfill detention dams. The power law relationship between the Darcy-Weisbach friction factor and Reynolds number (Eq.3) was calibrated using a non-linear optimization program. Results showed $a = 54.0$ and $b = -0.077$. The model has been verified and validated with meas-
ured data. The successive reservoir storages cause the routed outflow hydrograph to experience a greater reduction and lag time in reaching a safe peak flow magnitude. The model considers the reciprocal effects of reservoirs on each other. The sensitivity associated with the important model parameters shows that $d_{50}$ is the most important parameter affecting the routing process.

**Appendix**

The following symbols are used in this paper:

- $A, B, A', B'$ = Empirical coefficients;
- $a, b, a', b'$ = Empirical coefficients;
- $d$ = Diameter of rockfill particle;
- $d_{50}$ = Average size of the rockfill material;
- $D$ = A parameter that can be calculated from Equation (10);
- $f$ = Weisbach coefficient;
- $g$ = Acceleration due to gravity;
- $h$ = Hydraulic head;
- $H_1$ = Upstream water depth across the rockfill dam;
- $H_2$ = Downstream water depth across the rockfill dam;
- $i$ = Hydraulic gradient across the rockfill dam;
- $L$ = Base of the rockfill dam;
- $Q$ = Flow rate;
- $Q_i$ = Reservoir inflow rate;
- $Q_o$ = Reservoir outflow rate;
- $Re$ = Reynolds’ number;
- $S$ = Reservoir storage;
- $t$ = time;
- $V$ = Velocity;
- $x$ = The longitudinal direction of the dam;
- $w$ = dam width (perpendicular to flow direction).

- $\alpha$ = Coefficient;
- $\beta$ = The angle of the upstream or downstream of the dam face relative to the horizontal direction;
- $\beta_1$ = The angle of the upstream face of the dam with the horizontal direction;
- $\Delta Q\%$ = The percentage of the difference between the peak flows of the last outflow hydrograph and the first inflow hydrograph relative to the first inflow hydrograph peak;
- $\Delta T\%$ = The time percentage difference between the time to peak of the last outflow hydrograph and the first inflow hydrograph relative to the first inflow hydrograph peak;
- $\sigma$ = Standard deviation of rock material;
- $\nu$ = Kinematic viscosity.

**REFERENCES**


**Table 1. Sensitivity of the parameters on $\Delta Q\%$ and $\Delta T\%$.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$L$ (m)</th>
<th>$d_{50}$ (m)</th>
<th>$\beta$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter range</td>
<td>25 to 100</td>
<td>0.025 to 0.1</td>
<td>45 to 90</td>
</tr>
<tr>
<td>$\Delta Q%$ changes</td>
<td>-53.18 to -64.42</td>
<td>-65.17 to -50.94</td>
<td>-53.73 to -63.11</td>
</tr>
<tr>
<td>$\Delta T%$ changes</td>
<td>69 to 106.4</td>
<td>100 to 63</td>
<td>78 to 103</td>
</tr>
</tbody>
</table>