

## **Estimation of Irrigation Water Demand Function, Analyzing its Cross and Symmetrical Relations with other Inputs (Qazvin Plain)**

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### **ABSTRACT**

Water is the most important input used in agriculture. Due to the scarce water resources and dry and semi-arid climatic conditions of Iran, water demand management has special importance in the whole economy, including the agricultural sector, because this sector has the largest share in water consumption. The purpose of this study was to estimate the water demand function and to analyze the cross and symmetrical relationships between water and other inputs. For this purpose, the Ordinary, Allen, and Morishima's substitution elasticity were calculated, and the substitution and complementary relationship between water and other inputs were determined. These elasticities determine the amount and sign of cross relationship of water. In order to achieve the objectives of research, the translog cost function, along with the input share equations were estimated using iterative seemingly unrelated regressions. The information was related to crops and period (2007-2015) in Qazvin. The results showed that water was a low-elasticity input and its value was -0.75. Also, the cross elasticity with pesticide, labor, machinery and land was calculated as 0.71, 0.99, 0.93, and 0.89, respectively, which implied the substitution relationship. Investigating symmetry of elasticities also implies the asymmetry of Ordinary and Morishima elasticities and symmetry of Allen's elasticity with other inputs. In this regard, the cross elasticity of inputs of pesticide, labor, machinery, land and water were calculated as 0.28, 0.86, 0.91, and 0.90, respectively, indicating the asymmetry of this elasticity. Differences between levels of cross elasticities depend on the cost share of the two inputs and the sign of estimated coefficient.

**Keywords:** Iterative seemingly unrelated regressions, Substitution Elasticity.

### **INTRODUCTION**

Climate diversity has provided favorable conditions for the cultivation of various tropical and temperate crops in different parts of Qazvin Province, so that various kinds of wheat, barley, alfalfa, saffron, sugarbeet, lentils, beans, potatoes and vegetables are cultivated in a wide range of farms in Qazvin Province. Based on the results of the agricultural general census of 2014, 77,360 agricultural units are at least engaging in one of the agricultural activities, including farming. According to the results of this census in the same year, from 370,000 hectares of agricultural lands, 313,000

hectares are annual crops and 57,000 hectares are gardens/orchards. Farms are on a vast and fertile plain with an area of 13,000 square kilometers, which is considered as one of the most important agricultural centers in Iran (Statistical Yearbook of Qazvin, 2012). Therefore, due to the position of the agricultural sector as one of the factors of economic growth and food providing for a growing population, agricultural production must be increased. Increase in production requires consumption of more inputs, including water. Despite the fact that drought has long been one of the enemies of Iran's agriculture, the average rain in Qazvin is between 200-300 mm per year. This plain's

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agricultural potential attracted the attention of domestic and foreign experts of water resources in 1962, and the recognition and then exploitation of water in this plain began. Experts announced that the potential of the plain is 500 million  $\text{mm}^3$ , and now, after years of its exploitation, more than 1.5 billion  $\text{mm}^3$  of water are discharged, which is much higher than its potential and has led to a drop in groundwater levels and water deficit. According to the water deficit of 320 million  $\text{mm}^3$  and the volume of discharge, the potential of plain is estimated at 1.2 billion  $\text{mm}^3$  (Statistical Yearbook of Qazvin, 2012). According to statistics, more than 80 percent of Iran's water resources are dependent on groundwater resources, which accounts for 60 percent of the water needed in agriculture. Of the total of water utilized annually in the country, about 94% is used in agriculture, 5% in the household and 1% in the industrial sector (Office of Applied Water Resources Research, 2014). Although water plays a very

important role in agriculture, it has lower cost share than other inputs in this sector. Therefore, estimation of crops water demand and determining the relation of this input to other ones is required. In all studies in Iran (Table 1), water demand function is estimated for one crop.

However, in this research, we aimed to estimate the water demand function of all crops in Qazvin and, for all elasticity, the relationship between water and other inputs has been proved in appendix.

## MATERIALS AND METHODS

Basically, one can use two methods for extraction of input demand functions: 1. production function; and 2. cost function. The estimation of production function is more appropriate when value of the product is determined endogenous, while the cost function is preferred for the exogenous

**Table 1.** Selection of local and other studies on water econometrics and profit function.

Author	Research title	Method used
Stiroh (1999)	Measuring Input Substitution in Thrifts: Morishima, Allen-Uzawa, and Cross-Price Elasticities.	Econometrics (Profit function, SURE)
Schoengold <i>et al.</i> (2004)	Estimating the Panel of Agricultural Water Demand Function.	Econometrics (Profit function)
Zhou and Tol (2005)	Estimation of water demand function in agricultural, industrial and household sectors in China.	Econometrics (Profit function)
Ehsani (2009)	Estimation of economic value of water in autumn cereals in Qazvin.	Econometrics (Profit function)
Sadeghi <i>et al.</i> (2012)	Estimation of water demand function of tomato.	Econometrics (Profit function)
Azamzadeh Shouraki <i>et al.</i> (2013)	Effects of declining energy subsidies on value added in agriculture.	Econometrics (Profit function)
Falahi <i>et al.</i> (2013)	Extracting demand functions and determining the economic value of water in the production of major crops in Sydan-Farooq Plain.	Econometrics (Profit function)
Sobhani and Manzur (2014)	Estimation of energy and energy substitution in the chemical industry of Iran.	Econometrics (Profit function, SURE)
Golzari <i>et al.</i> (2015)	Estimation of water economic value in wheat production in Gorgan City.	Econometrics (Profit function)
Griessbach <i>et al.</i> (2015)	Estimating the Panel of Agricultural Water Demand Function.	Econometrics (Profit function)
Sun <i>et al.</i> (2017)	Estimation of Irrigation Water Demand Function and Water Economic Returns in China.	Econometrics (Profit function)

amount of production (Kant and Nautiyal, 1997). Use of the cost function is more efficient than production function because: (1) Cost is a function of the price of inputs. There is a very low multicollinearity between input prices, while there is a high degree of multicollinearity between the consumption of inputs, (2) The cost is a homogeneous function in relation to the price and there is no need to impose a degree of homogeneity unlike the production function, (3) In cost function, prices are assumed to be exogenous, whereas in production function, level of the inputs are considered exogenous. Considering this fact that decision making of managers of the agricultural unit is based on inputs, the use of cost function is more appropriate, and (4) Cost function, based on duality theorem between production and cost, has all the characteristics of production function (Onghena *et al.*, 2014).

The Translog Cost Function and Cost Share Equations of Inputs

The translog cost function was first introduced in 1971 by Sen, Jorgenson and Lao, and its general form. The logarithmic form of the translog cost function is as follows:

$$\begin{aligned} \ln \text{Cost}(p, VQ) = & c + 0.5c_{vqvq} \sum_{i=1}^n (\ln VQ)^2 + c_{vq} \sum_r \ln VQ + \\ & c_{pi} \sum_r \ln(p_j) + 0.5c_{pij} \sum_{i=1}^n \sum_{j=1}^n \ln(p_i)(p_j) + \\ & 0.5a_{piVQ} \sum_{i=1}^n \sum_{r=1}^m \ln(p_i) \ln(VQ) \end{aligned} \quad (1)$$

In the Eq. 1, Cost ( $p, VQ$ ) is the total cost of production,  $VQ$  is the value of product production,  $p_i$  is the price of  $i$ th input and  $c$ .

$c_{vq}$ ,  $c_{vqvq}$ ,  $c_{pij}$  and  $c_{pvq}$  are the cost parameters of the translog cost function that must be estimated. According to the Shepherd's Lemma, the demand function is obtained by deriving the cost function relation to the price of the inputs used in the agricultural sector (Berndt, 1991):

$$de_i = de(p_i, VQ) = \frac{\partial \ln \text{Cost}(p, VQ)}{\partial \ln(p_i)} \quad (2)$$

The demand function of  $i$ th input is equal to the cost share of each input from the total cost of production:

$$Sh_i = \frac{P_i C_{0i}}{\text{Cost}(p, VQ)} = de_i = de(p_i, VQ) \quad (3)$$

$$\sum_{i=1}^N Sh_i = 1 \quad (4)$$

Where,  $C_{0i}$  is the consumption of input and  $Sh_i$  is the cost share of input. In addition, the sum of share cost of all inputs should be equal to 1 according to Equation (4). In order to estimate the translog cost function, a set of constraints must be imposed on it so that it may be considered as a well-behaved cost function. These assumptions include: (1) Assumption of symmetry (Henderson and Quandt, 1961); (2) Assumption of the homogeneity of the production function: the production level and the cost share of inputs are independent from each other; (3) Assumption of existence of a Cobb-Douglas function, and (4), Fixed returns to scale (Berndt, 1991).

### Elasticity

In order to investigate the relationship between inputs and the sensitivity of their prices to demand each other or the price sensitivity of an input to its demand, the concept of substitution elasticity is used. In this regard, three types of elasticity are introduced:

1- Ordinary price elasticity: It is the percentage of change in the demand of  $i$ th input per one percent change in price of the  $j$ th input. This type of elasticity may be of an own or cross type and is calculated as follows:

$$\epsilon_{ii} = \frac{\partial \ln C_{0i}}{\partial \ln p_j} = \frac{\hat{c}_{ij}}{sh_i} + sh_j - 1 \quad (5)$$

$$\epsilon_{ij} = \frac{\partial \ln C_{0i}}{\partial \ln p_i} = \frac{\hat{c}_{ij}}{sh_i} + sh_j \quad (6)$$

2- Allen's substitution elasticity: The complementary and substitution relationship between inputs are well determined using Allen's Substitution elasticity. This elasticity, like the Ordinary one, has two types of own and cross that are measured as follows (Blackor *et al.*, 1977):

$$ALLEN_{ij} = (\hat{c}_{ij} + sh_i sh_j) / sh_i sh_j \quad (7)$$

$$ALLEN_{ii} = (\hat{c}_{ij} + sh_i (sh_i - 1)) / sh_i^2 \quad (8)$$

3- Morishima's substitution elasticity: This elasticity is a more complete criterion for assessing the elasticity among production



factors, compared with the first and second types. It has been defined as the percentage change in the ratio of the two inputs demand (i and j) into the percentage change in the price of one of the inputs. There is no own elasticity in this type and it is calculated as follows (Blackor *et al.*, 1977):

$$MORISHIMA_{ij} = \frac{d(\ln Co_i / \ln Co_j)}{d \ln p_j} = \hat{c}_{ij} (sh_i^{-1} - sh_j^{-1}) + 1 \quad (9)$$

### Elasticity and Symmetry

As explained previously, one of the assumptions about the translog cost function is symmetry. This assumption is true for the coefficients of the cost function but we should study it in relation to elasticity. A concise glance at the computational formulas associated with the elasticity results in that the symmetry applies only to Allen's cross elasticity:

$$\epsilon_{ij} \left[ = \frac{\hat{c}_{ij}}{sh_i} + sh_j \right] \neq \epsilon_{ji} \left[ = \frac{\hat{c}_{ji}}{sh_j} + sh_i \right] \quad (10)$$

$$MORISHIMA_{ij} \left[ = \hat{c}_{ij} (sh_i^{-1} - sh_j^{-1}) + 1 \right] \neq MORISHIMA_{ji} \left[ = \hat{c}_{ji} (sh_j^{-1} - sh_i^{-1}) + 1 \right] \quad (11)$$

### Iterative Seemingly Unrelated Regression Method

If an econometric model is composed of several equations and each of them has different dependent and independent variables, but we use the same data for system estimation, then it is possible that the error term in one equation depends on an error term in another equation. In this case, one of the classic assumptions about the existence of a covariance of zero in the error term would be violated and the use of the least square's method will not be effective. Therefore, it is necessary to choose a method that considers the dependence and correlation between the disturbance terms. The seemingly unrelated regression method, which was first introduced by Zellner, 1962, is suitable among all system estimators. Assume that there are two regression equations - with dependent variable's vector  $Y_i$ , disturbance term of  $U_i$  and with dimensions of  $T \times 1$ ,  $U_i \sim (0, \partial_{ii} I_T)$  and explanatory variable

vector ( $X$ ) with dimensions of  $T \times K_i$  (Zellner, 1962):

$$Y_i = X_i \beta_i + U_i \quad (12)$$

We can sum up two equations in an equation in the following form:

$$Y = X\beta + U \quad (13)$$

Where,  $U$  and  $y' = (y'_1, y'_2)$  are vectors with dimensions of  $2T \times 1$  and  $X$  vector with dimensions of  $2T \times (K_1 + K_2)$  and  $\beta$  is a vector with dimensions of  $(K_1 + K_2) \times 1$ . The matrix of the disturbance term has a framework as the following:

$$\Omega = \begin{bmatrix} \partial_{11} I_T & \partial_{12} I_T \\ \partial_{21} I_T & \partial_{22} I_T \end{bmatrix} = \Sigma \otimes I_T, \Sigma = [\sigma_{ij}] \quad (14)$$

In Equation (14),  $i$  and  $j$  are indices of the equation numbers. If the vector of  $X_i$  is the same in both of the equations, as  $X_1 = X_2 = X^* K_1 = K_2$ , then, estimators of the generalized least squares method in seemingly unrelated regressions will be exactly equivalent to ordinary least squares regression estimators (Zellner, 1962):

$$X = I_2 \otimes X^* \quad (15)$$

$$\begin{aligned} \widehat{\beta}_{GLS} &= \\ & [(I_2 \otimes X^*) (\Sigma^{-1} \otimes I_T) (I_2 \otimes X^*)]^{-1} [(I_2 \otimes X^*) (\Sigma^{-1} \otimes I_T) y] = \\ & [\Sigma \otimes (X^* X^*)^{-1}] [(\Sigma^{-1} \otimes X^*) y] = \\ & [I_2 \otimes (X^* X^*)^{-1} X^*] y = \widehat{\beta}_{OLS} \end{aligned} \quad (16)$$

In the translog cost function, the equations include the cost function and the cost share equations of inputs. If the number of inputs is  $i$ , then the number of equations will be also  $i$  (the cost function and  $(i - 1)$  the share-cost equation):

$$TC = \text{Cost}(p_i, VQ) \quad (17)$$

$$Sh_i = \frac{P_i Co_i}{\text{Cost}(p, VQ)} = sh(p_1, VQ) \quad (18)$$

## RESULTS AND DISCUSSION

Considering that the data were related to the period of (2007-2015), stationary of the variables were investigated in order to prevent spurious regression. As shown in Table 2, all variables were examined by the Phillips-Perron (PP) test. Phillips-Perron (1988) developed a number of unit root tests. The null hypothesis states that a variable has a unit root and the opposite hypothesis states

**Table 2.** Test for determining the static variables. <sup>a</sup>

Variables	Probability	Phillips-Perron	Result
Production value	0.00	117.44***	At level (With intercept)
Water	0.001	57***	At level (With intercept)
Land	0.00	106.72***	At level (With intercept)
Machinery	0.00	142.39***	At level (With intercept)
Labor	0.00	63.32***	At level (With intercept)
pesticide	0.00	129.52***	At level (With intercept)
Share water cost	0.00	81.99***	At level (With intercept)
Share of land cost	0.003	64.18***	At level (With intercept and trend)
share of machinery costs	0.002	77.42***	At level (With intercept and trend)
Share of labor cost	0.001	79.76***	At level (With intercept and trend)
Share of cost of pesticide	0.001	77.8	At level (With intercept)

<sup>a</sup> Source: Research findings. \* Significant at the level of 15%, \*\* Significant at the level of 10%, and \*\*\* Significant at the level of 5%.

that a variable has stationary state. The results showed that the variables of production value, price of water, price of land, price of machinery, price of pesticide and cost share of water did not have unit root at level and with intercept. Also, variables of cost share of labor, pesticide, land and machinery lacked unit root at level and with intercept and trend variable. Because of the two-dimensional nature of the information, the F-Limer test was conducted to determine the type of data. As the results of Table 3 show, the null hypothesis of this test was rejected based on the pool model and the model with panel data is accepted.

After determining the type of model and selecting the panel data, Hausman test was used to test the type of random or fixed effects. According to Table 3, the null hypothesis was accepted with a probability of 0.15%, indicating that there is a random effect.

### Estimation of the Experimental Model

The results of estimating the model and indicators related to the coefficients are presented in Table 4. Also, in the method of

estimating the cost function along with the cost share equations, elasticity is used in order to determine the relationship between inputs. We will describe it in more detail in the followings. In order to avoid dependence between cost share equations, all price variables were introduced into the model in relative terms. In this research, all prices were divided by the price of the pesticide input, having the lowest cost share, and then its equation of share cost was eliminated from the set of equations. Then, translog cost function, along with a set of equation of cost share (labor, land, water and machinery), was estimated systematically using iterative seemingly unrelated regression models. Table 4 shows the results of the estimation, and that all coefficients became almost significant at levels 1%, 5%, and 10 percent. The insignificance of coefficients can be due to the symmetry of the hessian matrix of the total cost in the cost-share equations. Of course, these coefficients cannot be interpreted on their own, and they are used to calculate elasticity.

**Table 3.** Determining the type of data: pool or panel. <sup>a</sup>

Test	Null hypothesis	Probability	Result
F Limer	pool	0.001	Acceptation of panel data
Hausman	random effects	0.15	Acceptation of the null hypothesis

<sup>a</sup> Source: Research findings.

**Table 4.** Results of model estimation. <sup>a</sup>

Coefficient	Level	Probability
Intercept	4133.35	0.628
Labor	0.35***	0.00
Machinery	0.22***	0.00
Water	0.19***	0.00
Land	0.17***	0.00
Production value	2.36***	0.00
The second power of labor	0.0002**	0.1
Labor, machinery	-0.0004**	0.1
Labor, water	-0.0002	0.58
Labor, land	-0.00005***	0.00
Labor, production value	0.002***	0.006
The second power of machinery	0.06*	0.1
Machine, water	-0.006	0.39
Machinery, land	-0.0004*	0.06
Machine, production value	0.00003	0.3
The second power of water	0.015	0.31
Water, land	-0.00002	0.86
Water, production value	0.00005	0.52
The second power of land	-0.0009	0.89
Land, production value	0.000067	0.73
The second power of production value	-0.000014***	0.00

<sup>a</sup> Source: Research finding. \* Significant at the level of 15%, \*\* Significant at the level of 10%, and \*\*\* Significant at the level of 5%.

After estimating the model, the coefficients of the price of pesticide, which had been eliminated from the equation of share, were calculated based on the parameters obtained in relation to other inputs. The parameters related to this input are presented in Table 5, which include pesticide price (0.51), cross-correlation coefficient between the pesticide and labor price (0.0005), cross-correlation coefficient of the pesticide and machinery price (0.004), cross-correlation coefficient of pesticide and water price (-0.007), Cross-correlation coefficient of pesticide and land price (0.0001), and square of the pesticide price (0.008).

In order to make translog cost function well-behaved, some limitations should be imposed on it. The previously mentioned limitations were tested. The results showed that the assumption of the homogeneity of the production function with the probability of 0.04 at level of 5% was rejected. The assumption of constant return to scale the Cobb-Douglas production function was also rejected and the translog form of the cost

function and the homogeneity of the cost function relative to the price level and the symmetry of its coefficients at the 5% level was accepted.

### Water Demand Function

Water demand function is formulated as follows:

$$x_W^d = f(VQ, P_L, P_M, P_W, P_{La}, P_p) \quad (19)$$

In this formula, we have  $x_W^d$ , amount of water demand in the agriculture sector in terms of million cubic meters,  $VQ$ , value of farming products in terms of million Rials,  $P_L$ , wage of the farm labor,  $P_M$ , cost of using machinery at farm in terms of million Rials per hectare,  $P_W$ : cost of consumed water in terms of millions of Rials and  $P_{La}$  the rent of farming land in terms of millions of Rials. After estimating the model by iterative seemingly unrelated regressions, the experimental model of the water demand function was obtained as follows:

**Table 5.** Calculation of the coefficients for the input with the lowest cost share. <sup>a</sup>

	Coefficient	Calculated level	Formula of calculating coefficient
Pesticide		0.051	$1 - \sum_{i=1}^{N-1} c_{pi}$
Pesticide, labor		0.0005	$-\sum_{i=1}^{N-1} c_{pi(j=2)}$
Pesticide, machinery		0.0004	$-\sum_{i=1}^{N-1} c_{pi(j=3)}$
Pesticide, water		-0.007	$-\sum_{i=1}^{N-1} c_{pi(j=4)}$
Pesticide, land		0.0001	$-\sum_{i=1}^{N-1} c_{pi(j=5)}$
The second power of pesticide		0.008	$-\sum_{j=1}^{N-1} c_{p(i=1)j}$

<sup>a</sup> Source: Research findings.

$$x_W^d = 0.19 + 0.14(VQ) - 0.0001(P_L) - 0.006(P_M) - 0.015(P_W) - 0.001(P_{La}) - 0.13(P_p) \quad (20)$$

The coefficient of water price in the above formula is -0.015, which indicates a negative relationship between the amount of water demand and its price, and is in accordance with economic theories. We will examine the relationship between the input of water and other inputs in the following.

### Types of Elasticity and Symmetry Checking

To examine the relationship between inputs, the estimation of coefficients between inputs was not sufficient. Therefore, Ordinary, Allen and Morishima's (own-cross) price substitution elasticities were calculated in Table (6):

As the results show, the amount of ordinary's own-price elasticity is less than one and negative for all the inputs and is consistent with the assumptions of economic theories. The amount of the rate of inputs for all inputs including pesticide, labor, machinery, water and land was calculated as -0.27, -0.71, -0.75, -0.85, and -0.86, respectively. The negativity of own-price

elasticity of inputs implies the rule of demand. Elasticity values less than 1 show low level of sensitivity. In other words, one percent change in the input prices causes farmer's demand for inputs to be less than one unit, and vice versa.

The value of Ordinary cross elasticity is positive and more than one for all the inputs, except for water and pesticide. This positivity shows the substitution relationship between the inputs. This relationship denotes that if the price of a substitute input increases (decreases) by 1 percent, the amount of its demand by the farmer will increase (decrease) less than 1 percent. The mutual relationship between two inputs of water and pesticide are of complimentary type, which has a meaning contrary to the substitution relationship. In this relationship, a one percent increase (decrease) in the price of an input causes decreasing (increasing) of the complimentary input. The factor determining whether this change is more or less, is the amount of cross elasticity. In this research, this value was calculated less than 1, therefore, with a one percent increase (decrease) in the price of water, the farmer's demand for the pesticide will decrease (increase) less than one percent.

**Table 6.** Ordinary, Morishima, and Allen's own–cross price substitution elasticities. <sup>a</sup>

Input	Ordinary	Allen	Morishima
Labor	-0.71	-2.44	-
Machinery	-0.79	-4.62	-
Water	-0.75	-4.83	-
Land	-0.85	-5.86	-
pesticide	-0.27	-24.45	-
Labor, machinery	0.17	0.029	0.88
Labor, water	0.15	0.084	0.86
Labor, land	0.14	0.021	0.85
Labor, pesticide	0.3	0.08	0.57
Machinery, water	0.12	0.018	0.91
Machinery, land	0.14	0.021	0.94
Pesticide, machinery	0.2	0.03	1.002
Water, land	0.14	0.021	0.89
Pesticide, water	-0.47	-0.07	0.28
Pesticide, land	0.15	0.02	1

<sup>a</sup> Source: Research findings.

The results of the calculations of Allen's own and cross price elasticity leads to results similar to those of Ordinary elasticity. However, the absolute value of this index is more than one, because this elasticity is greater than the Ordinary one. Regarding the value of cross elasticities in this case, it should be mentioned that the own elasticity of all the inputs less than one and the cross elasticity of inputs of water and pesticide, like the previous status, has become negative, sign's positivity shows substitute relationship of the inputs and sign's negativity is a complementary relationship of them.

The Morishima cross substitution elasticity was also calculated. The results showed that the sign of this elasticity was positive for all

the inputs – approving substitution of all the inputs of production. On the other hand, the numeral value of this index was positive for inputs of machinery and pesticide, denoting a significant substitution relationship between these two inputs. Table 7 presents the symmetry of Ordinary and Morishima cross elasticity for all inputs.

Comparing Tables 7 and 6 shows that the value of the elasticities is different from each other. For instance, cross elasticity of both inputs of (pesticide, water) have negative signs but the absolute value of the elasticity of (pesticide, water) is greater than that of (water, pesticide). In other words, the type of substitution has not changed and we only face with change in the amount of substitution.

**Table 7.** Investigating the symmetry of Ordinary and Morishima substitution price elasticities. <sup>a</sup>

Input names	Morishima	Ordinary
Machinery, labor	0.99	0.28
Water, labor	0.99	0.29
Land, labor	0.99	0.29
Pesticide, labor	0.57	0.30
Water, machinery	0.93	0.13
Land, machinery	0.96	0.15
Machinery, pesticide	0.80	0.01
Land, water	0.90	0.14
Water, pesticide	0.71	-0.03
Land, pesticide	0.86	0.01

<sup>a</sup> Source: Research findings.



Whether an elasticity is large or small depends on the cost share of the input whose demand is being studied. If its cost share is less than that of another input, the amount of Ordinary cross elasticity will be more. As shown in Table 1, the share cost of water is higher than that of the pesticide, so, the Ordinary cross elasticity of water in the face of the changes of pesticide cost is smaller than the Ordinary cross elasticity of pesticide in the face of water price change ( $|-0.47| > |-0.03|$ ). In terms of the value of Morishima substitution elasticity, the matter is a bit more complicated. Thus, if the share cost of an input whose demand is being examined is more than the input whose price is changing, with the assumption that the cross-estimating coefficient between the two inputs derived from estimation of translog cost function is negative, its Morishima cross elasticity will also be greater. For instance, the value of cross elasticity of water in the face of the changing of pesticide price, will be more than Morishima cross elasticity of pesticide in the face of water price changes ( $|0.71| > |0.28|$ ).

### CONCLUSIONS

The purpose of this study was to estimate the water demand function in the agricultural sector of Qazvin Plain and determine the relationship between inputs used by farmers using information about different crops in the period 2007-2015. The results showed that water in this sector has low elasticity and its sign is in accordance with the principles of microeconomics. The own elasticity of other inputs is also less than one and has a negative sign. The highest elasticity belonged, respectively, to the inputs of land, machinery, and water. In other words, for a one percent change in the price of all inputs, the farmer's sensitivity to change in demand for land will be the most, and the sensitivity to changing demand for pesticide will be least of all. Since the Morishima elasticities have a more exact analysis of the relationship between production inputs, the values of this elasticity

were also investigated. The results showed that water had a substitution relationship with other inputs. The input of water had the highest substitution relationship with labor, that is, with the increase of labor price, the farmers' demand for water will be more than the increase of water demand as a result of the increase in price of other inputs. Minimum elasticity of water demand belongs to the pesticide price. Therefore, values of Morishima elasticity can be used for saving water. In other words, the greatest saving in water consumption will be achieved through decreasing the price of labor. According to the results of the research, the following suggestions are presented: Given the low elasticity of water, pricing policies have little effect on modifying the pattern of consumption and saving water. Therefore, we have to use alternative policies such as decreasing the price of the substitution inputs of water and choose the input that has the highest amount of substitution relationship with water. However, the prices of inputs are exogenous for farmers, but through reducing or increasing the subsidy by the state, the farmer's payment for each unit of input can be reduced. For example, if the objective is to save water, according to Morishima substitution price elasticity between water and machinery (0.93), reducing one percent of payment for machinery causes saving of 0.93% water consumption. The use of advanced irrigation technologies is very important. For the implementation of this policy, it is necessary to provide farmers with the opportunity to use the latest irrigation techniques.

### Appendix

1- Investigating the relationship between  $(\epsilon_{ij}, \epsilon_{ji})$ .

$$\text{If } Sh_i > Sh_j: \quad [1 - 1]$$

$$\epsilon_{ij} = \frac{\hat{c}_{ij}}{sh_i} + sh_j, \epsilon_{ji} = \frac{\hat{c}_{ji}}{sh_j} + sh_i \quad [1 - 2]$$

And assuming that the translog cost function is symmetric:



$$\begin{aligned} \hat{c}_{ij} &= \hat{c}_{ji} \quad [1 - 3] \\ \epsilon_{ij} - \epsilon_{ji} &= \left( \frac{\hat{c}_{ij}}{sh_i} + sh_j \right) - \left( \frac{\hat{c}_{ji}}{sh_j} + sh_i \right) \\ &= \left( \frac{\hat{c}_{ij} + sh_j}{sh_i sh_j} \right) - \left( \frac{\hat{c}_{ji} + sh_i}{sh_j sh_i} \right) \\ &= \frac{\hat{c}_{ij} + sh_j - \hat{c}_{ji} - sh_i}{sh_i sh_j} = \frac{sh_j - sh_i}{sh_i sh_j} \\ &\neq 0 \rightarrow \\ (\epsilon_{ij} - \epsilon_{ji}) &> 0 \text{ if } (sh_j - sh_i) > 0 \quad [1 - 4] \end{aligned}$$

As a consequence, in the study of the Ordinary substitution elasticity of the two inputs, if the price of the input with a lower cost share increases, then the amount of demand will change more than the situation when the input price changes with a larger share of costs, indicating the asymmetry of ordinary elasticity between two inputs.

2- Investigating the relationship between (MORISHIMA<sub>ij</sub>, MORISHIMA<sub>ji</sub>).

$$\text{a: If } Sh_i > Sh_j, (\hat{c}_{ij} = \hat{c}_{ji}) > \quad [2 - 1]$$

$$\text{0 : MORISHIMA}_{ij} = \hat{c}_{ij}(sh_i^{-1} - sh_j^{-1}) + 1 \quad [2 - 2 - 1]$$

$$\text{MORISHIMA}_{ji} = \hat{c}_{ji}(sh_j^{-1} - sh_i^{-1}) + 1 \quad [2 - 2 - 2]$$

$$\text{MORISHIMA}_{ij} - \text{MORISHIMA}_{ji} \quad [2 - 3]$$

$$= [\hat{c}_{ij}(sh_i^{-1} - sh_j^{-1}) + 1]$$

$$- [\hat{c}_{ji}(sh_j^{-1} - sh_i^{-1}) + 1]$$

$$= \hat{c}_{ij}(sh_i^{-1} - sh_j^{-1}) + 1$$

$$- \hat{c}_{ji}(sh_j^{-1} - sh_i^{-1}) - 1$$

$$= \hat{c}_{ij}(sh_i^{-1} - sh_j^{-1})$$

$$- \hat{c}_{ji}(sh_j^{-1} - sh_i^{-1})$$

$$= \hat{c}_{ij}[sh_i^{-1} - sh_j^{-1} - sh_j^{-1}$$

$$+ sh_i^{-1}] = \hat{c}_{ij}[2sh_i^{-1} - 2sh_j^{-1}]$$

$$= 2\hat{c}_{ij}[sh_i^{-1} - sh_j^{-1}]$$

$$=$$

$$\xrightarrow{Sh_i > Sh_j, Sh_i, Sh_j > 0 \rightarrow (1/Sh_j) > (1/Sh_i)}$$

$$2\hat{c}_{ij}[sh_i^{-1} - sh_j^{-1}] < 0, \neq 0$$

$$\begin{aligned} &\rightarrow \text{MORISHIMA}_{ij} \quad [2 - 4] \\ &< \text{MORISHIMA}_{ji} \end{aligned}$$

The result is that if the price of the input with a lower cost share is increased, and if the mutual estimation coefficient between the two inputs obtained from the estimated Translog cost function is positive, then the change in amount of the other input demand will be less than the situation where the price changes with a higher cost share.

Now, if the mutual estimation coefficient between the two inputs is negative, then we have:

$$\text{b: If } Sh_i > Sh_j, (\hat{c}_{ij} = \hat{c}_{ji}) < 0 : \quad [2 - 5]$$

$$\text{MORISHIMA}_{ij} - \text{MORISHIMA}_{ji} \quad [2 - 6]$$

$$= 2\hat{c}_{ij}[sh_i^{-1} - sh_j^{-1}]$$

$$\xrightarrow{Sh_i > Sh_j, Sh_i, Sh_j > 0 \rightarrow (1/Sh_j) > (1/Sh_i)}$$

$$= \frac{2\hat{c}_{ij}[sh_i^{-1} - sh_j^{-1}]}{sh_i sh_j} > 0, \neq 0$$

$$\rightarrow \text{MORISHIMA}_{ij} \quad [2 - 7]$$

$$> \text{MORISHIMA}_{ji}$$

In this case, if the price of the input that has a lower cost share is increased, then the amount of demand for the other will change more than the situation where the price will change with a larger share of the cost. Finally, the asymmetry of Morishima's substitution price elasticity is proved.

3 Investigating the relationship between (ALLEN<sub>ij</sub>, ALLEN<sub>ji</sub>).

The symmetry of Allen's elasticity between two inputs is easily verifiable:

$$\text{ALLEN}_{ij} \quad [3 - 1]$$

$$= (\hat{c}_{ij} + sh_i sh_j) / sh_i sh_j$$

$$\text{ALLEN}_{ji} \quad [3 - 2]$$

$$= (\hat{c}_{ji} + sh_j sh_i) / sh_j sh_i$$

$$\xrightarrow{Sh_i > Sh_j, Sh_i, Sh_j > 0} \text{ and } \xrightarrow{Sh_i > Sh_j, Sh_i, Sh_j > 0, (\hat{c}_{ij} = \hat{c}_{ji})} \quad [3 - 3]$$

$$\text{ALLEN}_{ij} - \text{ALLEN}_{ji} \quad [3 - 4]$$

$$= [(\hat{c}_{ij} + sh_i sh_j) / sh_i sh_j]$$

$$- [(\hat{c}_{ji} + sh_j sh_i) / sh_j sh_i]$$

$$= [(\hat{c}_{ij} + sh_i sh_j) - (\hat{c}_{ji} + sh_j sh_i)] / sh_i sh_j$$

$$= [\hat{c}_{ij} + sh_i sh_j - \hat{c}_{ji} - sh_j sh_i] / sh_i sh_j = 0$$

$$\rightarrow \text{ALLEN}_{ij} - \text{ALLEN}_{ji} = 0 \quad [3 - 5]$$

$$\text{ALLEN}_{ij} = \text{ALLEN}_{ji}$$

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## برآورد تابع تقاضای آب در بخش زراعی و تحلیل روابط متقابل و متقارن میان نهاده آب و سایر نهاده‌ها (دشت قزوین)

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### چکیده

آب از مهمترین و ضروری ترین نهاده‌های مورد استفاده در بخش کشاورزی می‌باشد. با توجه به محدودیت منابع آبی و شرایط اقلیمی خشک و نیمه خشک کشور، مدیریت تقاضای آب در کل اقتصاد و از جمله بخش کشاورزی که بیشترین سهم را در مصرف آب دارد، از اهمیت ویژه‌ای برخوردار است. هدف این مطالعه برآورد تابع تقاضای آب و تحلیل روابط متقابل و متقارن میان نهاده آب و سایر نهاده‌هاست که به این منظور کشتش‌های متقاطع معمولی، آلن و موریشیما محاسبه و رابطه جانشینی یا مکملی نهاده آب و میزان این رابطه با سایر نهاده تعیین شد. جهت تأمین اهداف تحقیق تابع هزینه ترانسلوگ به همراه معادلات سهم هزینه نهاده‌ها، با استفاده از روش رگرسیون‌های به ظاهر نامرتب تکراری برآورد گردید. اطلاعات مورد نیاز مربوط به محصولات زراعی دشت قزوین و دوره زمانی (۱۳۹۴-۱۳۸۶) می‌باشد. نتایج تحقیق نشان داد که در طول دوره مورد بررسی آب کالایی کم کشتش می‌باشد و میزان آن  $0/75$ - محاسبه شد، همچنین کشتش متقاطع موریشیمای این نهاده با نهاده‌های سم، نیروی کار، ماشین‌آلات و زمین به ترتیب  $0/93$ ،  $0/99$ ،  $0/71$  و  $0/89$  محاسبه گردید که دلالت بر رابطه جانشینی نهاده آب با سایر نهاده‌ها دارد. بررسی تقارن کشتش‌ها نیز دلالت بر عدم تقارن کشتش معمولی و کشتش موریشیما و تقارن کشتش آلن نهاده آب با دیگر نهاده‌ها بود. در این راستا کشتش متقاطع موریشیمای نهاده‌های سم، نیروی کار، ماشین‌آلات و زمین با نهاده آب، به ترتیب  $0/28$ ،  $0/86$ ،  $0/91$  و  $0/90$  حاصل شد که نشان‌دهنده متقارن نبودن این کشتش بود که میزان تفاوت کشتش‌ها بستگی به تفاوت در سهم هزینه دو نهاده مورد بررسی دارد.