

Evaluation of Farmer's Risk Attitudes Using Alternative Utility Functional Forms

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ABSTRACT

The most commonly used utility functional forms are reviewed and their properties compared. Then, using data collected from West Azarbaijan province, utility functions are estimated and absolute risk aversion coefficients measured. Exponential and exp-power utility functions classified all farmers as risk averse, but quadratic and cubic utility functions classified 75% and 65% of farmers as risk-averse, respectively. Findings in this study indicated that alternative utility functions may classify farmers' risk attitudes in different ways.

Keywords: Agricultural economics, Risk attitudes, Utility function.

INTRODUCTION

Risk aversion parameter estimation from farm level data constitutes a significant line of inquiry in the literature on applied risk [7]. There are different approaches to risk aversion parameter estimation, but the most commonly used approach is estimation by using a utility function. The choice of utility functional form is an important issue in decision analysis under the expected utility hypothesis and can affect classification of risk preferences [9]. The question that must be answered is: what kind of utility functional forms are appropriate? Pratt [6] argued that utility functions exhibiting decreasing absolute risk aversion (DARA) are logical candidates for use when trying to describe the behavior of people. In general, most researchers agree that continuity and decreasing the absolute risk aversion of a utility function are sufficient utility choice criteria. However, beyond that, there is little guidance for researchers to use in selecting functional forms. Many studies have arbitrarily chosen a particular functional form and then proceeded with the analysis of risk

attitudes [1,4].

Most of the available functional forms require certain restrictive a priori assumptions to be made, which might not be appropriate under some conditions. For example, decreasing absolute risk aversion has emerged as a stylized fact while the empirical evidence on this question is scant. While a substantial body of research has found evidence of risk aversion [see, for example, 1,3,4,8] it is unclear whether risk aversion decreases, stays constant or increases with wealth.

Lin and Chang [5] suggested using a Box-Cox transformation as a means of determining the form of utility function rather than simply assuming it. However, the Box-Cox transformation is not consistent with Bernoullian decision theory [2]. The appropriate utility function, however, must not only employ the least restrictive assumptions but must also be consistent with Bernoullian decision theory.

Zuhair, Taylor, and Kramer [9] studied the effects of the choice of utility functional forms on the classification of risk preferences and the prediction of farmer decisions. They elicited subjective utility values and

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probability distributions for price and yield from Sri Lankan producers of minor export crops and, finally, estimated the exponential, quadratic, and cubic utility functions. Their findings indicated that the choice of functional form affects both the classification of risk attitudes and the prediction of harvesting strategy.

Saha [7] has proposed a new utility function-which he calls 'expo-power'-that exhibits decreasing, constant, or increasing absolute risk aversion and decreasing or increasing relative risk aversion, depending on parameter values. Numerical analysis has suggested that the expo-power utility function performs well in incorporating these risk preference structures, while arbitrary risk preference specifications may lead to biased risk response estimates.

In this paper, the most commonly used utility functional forms were reviewed first and their properties compared. Then, using data collected from West Azarbaijan province, utility functions were estimated and absolute risk aversion coefficients were measured. Finally, the effect of functional form on risk aversion coefficients was discussed.

Utility Functional Forms

There are different forms of utility function for evaluating the relationship between income levels of a consumer or producer and his [or her] utility indices. In this section, the most commonly used utility functional forms are introduced.

1) Quadratic Utility Function:

Early researchers preferred the quadratic utility function:

$$U = a + bM + cM^2 \quad b > 0, c < 0$$

where U is utility and M is the money measure. The properties of this functional form are: (i) when combined with linear profit functions, it generates quadratic expected utility functions that are easily maximized using ordinary programming

routines; and (ii) it is easily fitted using OLS to utility questionnaire data.

The absolute risk aversion coefficient for quadratic utility function is:

$$R_a = -2c/(b + 2cM)$$

This coefficient rises with an increase in the money measure. In other words, an increase in wealth causes an increase in risk aversion, a conclusion which is not very realistic in actual world.

2) Cubic Utility Function

The cubic utility function can be presented as:

$$U = a + bM + cM^2 + dM^3$$

where a , b , c , and d are parameters. The second derivative is given by $2c + 6dM$, the sign of which depends on the sign and magnitude of the parameters c , d , and the level of the money measure, M . Thus, increasing and decreasing marginal utility are both possible. The Arrow-Pratt absolute risk aversion coefficient for cubic utility function is:

$$R_a = - \{ (2c + 6dM) / (b + 2cM + 3dM^2) \}$$

R_a can thus be either positive or negative depending on the parameter values and income (wealth) at which the equation of R_a is evaluated.

3) Exponential Utility Function

The exponential utility function for money has long attracted attention because it exhibits a non-increasing (in fact, constant) absolute risk aversion. Also, under certain conditions, it generates an expected utility function that is maximizable within a quadratic programming model. However, this functional form presents estimation problems. Logarithmic transformation of an exponential utility function does not conform to the von Neumann-Morgenstern axioms [9]. Hence, it cannot be used as a basis for best fit in statistical analysis.

The exponential utility function can be given as:

$$U = a - be^{-\lambda M} \quad \text{for } a, b, \lambda > 0$$

where e is the base for natural logarithms. The second derivative of this function is:

$$-\lambda^2 be^{-\lambda M} < 0$$

which means that the marginal utility of the function is diminishing. The Arrow-Pratt absolute risk aversion coefficient, R_a , is equal to λ , which is positive and constant. The exponential utility function, therefore, exhibits constant risk aversion over all levels of income, which can be argued as one of its major limitations.

Exponential utility function and normally distributed income, $M \sim N(\mu, \sigma^2)$, produce the following expected utility:

$$E[U(M)] = a - b \exp[-\lambda\mu + (\lambda^2/2)\sigma^2]$$

4) Expo-Power Utility Function

The expo-power utility function takes the form [7]:

$$U = a - \exp(-\beta M^\alpha) \quad \alpha \neq 0, \beta \neq 0, \alpha\beta > 0$$

This utility function has the following properties:

a) The Arrow-Pratt coefficient of absolute risk aversion is given by:

$$R_a = \{(1 - \alpha + \alpha\beta M^\alpha)/M\}$$

b) Under its parameter restriction, this function exhibits decreasing absolute risk aversion if $\alpha < 1$, constant absolute risk aversion if $\alpha = 1$, and increasing absolute risk aversion if $\alpha > 1$.

c) The expo-power utility function is quasi concave for all $M > 0$.

d) The necessary condition for (strict) concavity of expo-power utility function is given by

$$\alpha - \alpha\beta M^\alpha - 1(<) \leq 0$$

and sufficient condition is given by

$$\alpha(<) \leq 1.$$

Expo-power utility function is a flexible form and does not impose any predetermined risk preference structure on risk attitudes. In risk programming or simulation models, a utility function is directly specified or an optimization method adopted that lies on an underlying utility functional form, while most functional forms impose a specified risk preference structure. Nevertheless, the empirical evidence observed regarding the nature of absolute and relative risk aversion is very ambiguous, and the a priori reasons to assume a particular risk preference structure are therefore weak usually [7].

Thus, the expo-power utility function, which is free from risk preference restrictions, may be useful in providing the underpinning for risk programming and simulation models.

METHODOLOGY

The region where data was collected is located in Bookan district West Azarbaijan, where dry farming, particularly wheat and peas, is dominant with regard to climatic conditions. An essential component of dry farming is production risk, and so the evaluation of risk attitudes under such conditions has greater importance. In a stratified random sampling method, a uniform stratum of some farmers of the district (containing 200 farmers in ten villages) was first established on the basis of soil and water quality, level of mechanization, cropping pattern, type of cropping (dry farming), and the minimum education level of farmers (their ability in reading and writing). Then, a simple random sample was selected and 20 farmers were interviewed by questionnaire.

In the questionnaires, a comprehensive set of questions about farm inputs, outputs, costs and different activities was asked, information relating to income variation over the last five years was obtained and the range of the income variable was determined. Furthermore, the subjective utility of different levels of income was elicited by a direct elicitation method. The applied elicitation method was the Equally Likely Certainty Equivalent (ELCE). In the ELCE approach the decision-maker is asked to choose between two-state risky prospects with equal probability of 0.5 for each state. This method avoids bias caused by probability preferences, which could be confronted when using the ordinary von Neuman-Morgenstern (M-N) model. The ELCE method overcomes the criticism of a bias due to probability preferences. However, it still has the difficulty that the subject is forced to select between a certainty and a lottery. Nevertheless, this problem may be minimized by presenting the questions as



practical decision-making problems.

After the elicitation of subjective utilities for respondents, all four utility functional forms mentioned above were estimated for each farmer and the related absolute risk aversion coefficients were determined.

RESULTS AND DISCUSSION

To compare the absolute risk aversion coefficients in the alternative utility functional forms, the quadratic utility function ($U=a+bM+cM^2$), the cubic utility function ($U=a+bM+cM^2+dM^3$), the exponential utility function ($U=a-b\text{Exp}(-\lambda M)$), and expopower utility function ($U=a - \text{Exp}(-\beta M^\alpha)$) were estimated using the non-linear least square (NLS) method for all sample farmers. The initial values, where needed, were obtained from earlier studies. After testing and improving the statistical aspects of the data, the parameters of different utility functions were estimated and used for calculating risk aversion coefficients. In all cases, farmer subjective utility indices have a significant relationship with levels of farm income. However, a few parameters of the utility func-

tions were not statistically significant. For the quadratic utility function this occurred two times (b and c parameters for farmer number 9, and c parameter for farmer number 11); for the cubic utility function one time (c and d parameters for number 9); for the expo-power utility function four times (α and β parameters for farmer number 7, β parameter for farmer number 9, α and β parameters for farmer number 19, and α parameter for farmer number 20). For the exponential utility function, λ was significant in all cases. Using estimated parameters, the absolute risk aversion coefficients were obtained as presented in Table 1.

The quadratic utility function classified 15 farmers as risk-averse and five farmers as risk-preferring at the income midpoint. For risk-averse farmers, the R_a ranged from 0.007712 (farmer 19) to 0.000431 (farmer 7). For farmers classified as risk preferring, the R_a ranged from -0.00061 (farmer 1) to -0.01433 (farmer 17) at the midpoint of income.

The cubic utility function classified 13 farmers as risk-averse and 7 farmers as risk-preferring. The R_a for risk-averse farmers ranged from 0.006227 (farmer 8) to

Table 1. Absolute risk aversion coefficients using different utility functions.

Farmer Number	Quadratic Utility Function	Cubic Utility Function	Exponential Utility Function	Expo- Power Utility Function
1	-0.000610	-0.029310	0.0001793	0.001825
2	-0.002200	-0.006090	0.0001316	0.001129
3	0.002328	0.003710	0.0001920	0.002703
4	0.001195	-0.00083	0.0002241	0.001733
5	0.002164	0.000740	0.0003110	0.002778
6	0.001423	0.001056	0.0001213	0.002128
7	0.000431	-0.00020	0.0001682	0.001712
8	0.004070	0.006227	0.0004640	0.003846
9	-0.003030	-0.002890	0.0001482	0.001518
10	0.002752	0.001096	0.0003183	0.003145
11	0.004241	0.004314	0.0003381	0.003759
12	0.003413	0.001182	0.0003220	0.003704
13	-0.003890	-0.001080	0.0001306	0.001439
14	0.002578	0.001789	0.0005786	0.002959
15	0.002237	0.000217	0.0003063	0.002632
16	0.001380	0.002545	0.0002940	0.002146
17	-0.014330	-0.002170	0.0001203	0.001290
18	0.001584	0.000596	0.0002033	0.002387
19	0.007712	0.004215	0.0003229	0.006452
20	0.003663	0.004066	0.0018930	0.004405

Table 2. The ranking of absolute risk aversion coefficient using different utility functional forms.

No ^a	quadratic	No ^a	cubic	No ^a	exponential	No ^a	expo-power
19	0.007712	8	0.006227	20	0.0018930	19	0.006452
11	0.004241	11	0.004314	14	0.0005786	20	0.004405
8	0.004070	19	0.004215	8	0.0004640	8	0.003846
20	0.003663	20	0.004066	11	0.0003381	11	0.003759
12	0.003413	3	0.003710	19	0.0003229	12	0.003704
10	0.002752	16	0.002545	12	0.0003220	10	0.003145
14	0.002578	14	0.001789	10	0.0003183	14	0.002959
3	0.002328	12	0.001182	5	0.0003110	5	0.002778
15	0.002237	10	0.001096	15	0.0003063	3	0.002703
5	0.002164	6	0.001056	16	0.0002940	15	0.002632
18	0.001584	5	0.000740	4	0.0002241	18	0.002387
6	0.001423	18	0.000596	18	0.0002033	16	0.002146
16	0.001380	15	0.000217	3	0.0001920	6	0.002128
4	0.001195	7	-0.00020	1	0.0001793	1	0.001825
7	0.000431	4	-0.00083	7	0.0001682	4	0.001733
1	-0.000610	13	-0.001080	9	0.0001482	7	0.001712
2	-0.002200	17	-0.002170	2	0.0001316	9	0.001518
9	-0.003030	9	-0.002890	13	0.0001306	13	0.001439
13	-0.003890	2	-0.006090	6	0.0001213	17	0.001290
17	-0.014330	1	-0.029310	17	0.0001203	2	0.001129

^a No= Farmer number.

0.000217 (farmer 15) and for risk-preferring farmers, the R_a ranged from -0.00020 (farmer 7) to -0.029310 (farmer 1) at the income midpoint.

The exponential and expo-power utility functions classified all farmers as risk-averse. The R_a for exponential utility function ranged from 0.001893 (farmer 20) to 0.0001203 (farmer 17). In the exponential utility function, R_a is not related to income levels. For the expo-power utility function, the R_a ranged from 0.006452 (farmer 19) to 0.001129 (farmer 2) at the midpoint of income. The ranking of absolute risk aversion coefficients for alternative functions is presented in Table 2.

In summary, the findings of this study confirmed the results of previous studies and emphasized that the choice of utility function is an important aspect of risk attitude analysis. Alternative utility functions may classify farmers' risk attitudes in different ways. For example, while farmer 4 was classified as a risk-preferring farmer by the cubic utility function, he would be classified as risk-averse by other utility functions.

With respect to its theoretical properties, the expo-power utility function seems to be a better choice, since it is a flexible form and does not impose any predetermined risk preference structure on risk attitudes. In the present study, the expo-power utility function classified all farmers as risk-averse, which is quite consistent with previous evidence.

The estimated α parameter of the expo-power utility function for all farmers was positive and smaller than unity ($\alpha < 1$). Thus the farmers were DARA with an increase in the money measure. This result was also confirmed by R_a in the cubic utility function for farmers at different money levels.

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ارزیابی نگرش کشاورزان به ریسک با استفاده از فرمهای مختلف تابع مطلوبیت

ج. ترکمانی و م. حاجی رحیمی

چکیده

در این مقاله، ابتدا رایجترین فرمهای تابع مطلوبیت معرفی و خصوصیات آنها با هم مقایسه گردید. آنگاه در یک مطالعه موردی و با استفاده از نمونه‌گیری آماری انجام گرفته در منطقه بوکان، واقع در آذربایجان غربی، فرمهای مختلف تابع مطلوبیت برای کشاورزان مورد مطالعه بر آورد و معیار ریسک‌گریزی مطلق برای آنان محاسبه شد. نتایج حاصل از این مطالعه نشان داد که استفاده از فرمهای مختلف تابع مطلوبیت برای تخمین ضرایب ریسک‌گریزی می‌تواند رتبه‌بندیهای متفاوتی از میزان ریسک‌گریزی کشاورزان نشان دهد. حتی ممکن است یک کشاورز که با یکی از فرمهای تابعی ریسک‌گریز رتبه‌بندی می‌شود با یک فرم تابعی دیگر ریسک‌پذیر در نظر گرفته شود. تابع مطلوبیت نمایی-توانی تمام کشاورزان مورد مطالعه را ریسک‌گریز رتبه‌بندی کرد، اما توابع مطلوبیت درجه دوم و درجه سوم به ترتیب ۷۵٪ و ۶۵٪ کشاورزان را ریسک‌گریز معرفی نمودند.