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Positive Agricultural and Food Trade Model with *Ad Valorem* Tariffs

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ABSTRACT

Interest rate *Ad valorem* tariffs are considered as the most prominent trade tools extensively used in the framework of Spatial Equilibrium Models (SEMs) to analyze agricultural and food trade policies around the world. However the results obtained from such models have been criticized because of their inadequacy in producing any observed data within the base period. Hence a positive spatial and temporal trade model which incorporates *ad valorem* tariffs was developed throughout the ongoing study. The calibrated model helps researchers to perform a substantially flawless empirical trade study in the real world. A numerical example is finally presented at the end of the article to justify the findings of the model, and to compare welfare analysis of the calibrated *vs.* the uncalibrated model.

Keywords: *Ad valorem* Tariff, Agricultural Trade Model, Calibration, Spatial Equilibrium Model.

INTRODUCTION

This paper is a methodological work for the professional researchers in the field of agricultural and food products marketing and trade taking a different perspective on the traditional food trade models. The focal point of studies dealing with the movement of agricultural products and food among spatially separated firms and consumers is the SEM framework. SEM models basically consist of supply and demand functions for each region, and a network of the associated transportation costs. These kinds of models are originally attributed to Samuelson, Takayama and Judge (STJ) (Samuelson, 1952; Takayama and Judge, 1964). STJ approach maximized a quadratic quasi social gain function subjected to constraints satisfying supply and demand quantities in the regional markets. Numerous studies have surveyed theoretical as well as applied works on spatial modeling. As prominent studies, one can refer to the contributions of Martin (1981), Labys and Yang (1997), Hieu and Harrison (2011) and Devadoss, (2013). Based on these works, STJ modeling frameworks are proper to analyze various types of trade policy instruments.

Nevertheless, the empirical trade analysis based on SEM always have been suffered from the inaccuracy of exogenous parameters such as implicit trade cost as well as ill-measured supply and demand parameters. These imprecisions which have received less attention in the literature above lead to discrepancy between the equilibrium solution of the models and the observed information. These lack of accords have caused the trade policy evaluation of these imprecise models to become distorted. Such inaccuracies are typically the result of poor quality and not reliable trade data for estimating implicit trade cost as well as

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proper supply and demand parameters especially within the developing countries.

Numerous efforts have been made to solve the problem, for example Bouamra-Mechemache *et al.* (2008), Jansson *et al.* (2009), Paris *et al.* (2009) and Nolte *et al.* (2010) endeavored to develop a calibrated model. These attempts were either trivial or needed abundant information to make a model replicating the observed prices and quantities. Therefore due to a lack of a comprehensive calibrated model in the empirical works, this study was built up to develop a positive trade model based on the existing literature. The suggested model also incorporates *ad valorem* tariffs as an important trade instruments extensively used in the international trade research in the real world. In this regard, a temporal and spatial calibrated trade model was developed to simulate the results of *ad valorem* tariff policies throughout the market. The proposed model perfectly reproduced the observed supply and demand quantities as well as prices and trade flows, for a given base year, by attaching linear cost terms to storage and trade flows and by using all the available information.

To make the perception of the mathematical formulations easier, a diagrammatic explanation of SEM is required. So the remainder of this paper is structured as follows: First, a graphical presentation of SEM is depicted. Later, the mathematical statement of the models, and the calibration procedures are presented. The succeeding section reveals some computational results, including a numerical example with comprising two commodities, two regions and two time periods (2×2×2 model). Finally, the article is concluded with some suggestions for future works in the area.

A Graphical Presentation of SEM

Following Martin (1981), and Takayama and Judge (1964), Figure 1 provides the graphical representation of a two-region, one-commodity SEM model as conceived by Samuelson (1952). The model is illustrated in the standard quantity domain (primal form) by setting the prices as function of quantities. Since the regional demand and supply equations are often estimated with quantities as dependent variables, Martin (1981) has also shown the SEM models in a price domain (dual form), not discussed here. Figure 1 shows the two regions in the first and third graphs respectively, (called Regions 1 and 2), while the central graph represents the trade conditions, assuming known linear supply $P_i^S = f(Q_i^S)$ and demand $P_i^D = f(Q_i^D)$ functions in which the subscripts *i, j*= 1, 2 (i alias j) refer to regions. Without trade, autarky market equilibrium occurs at the intersection of the demand and

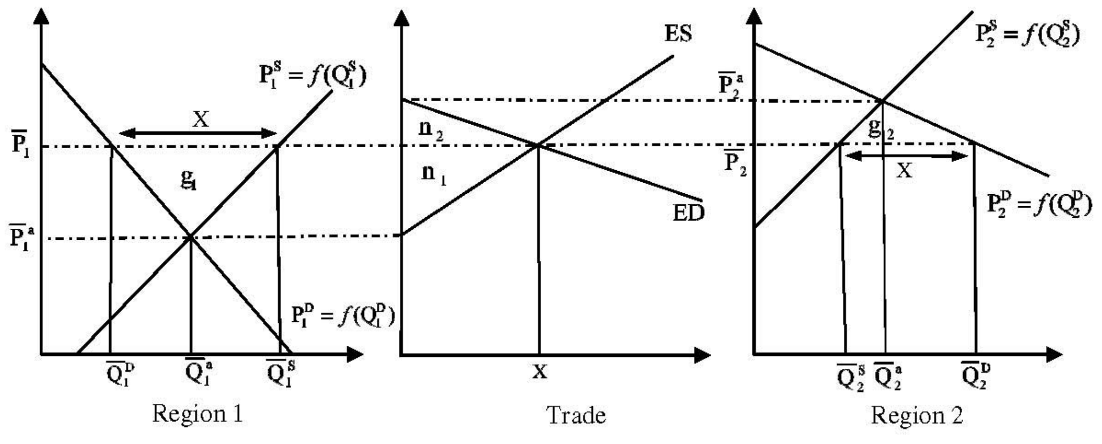

Figure 1. Spatial price equilibrium model.

supply of each region, where $\overline{P}_i^a$ and $\overline{Q}_i^a$ stand for the equilibrium price and quantity respectively.

The process of spatial arbitrage would occur if trade is allowed. The commodity flow would be from Region 1, where the autarky equilibrium price is lower than that in Region 2. An Excess Supply curve (ES) for Region 1 can be defined as the horizontal distance between demand and supply curves in that region, above the autarky equilibrium price $\overline{P}_1^a$. Similarly, for Region 2, an Excess Demand curve (ED) can be defined by taking the horizontal distance between the supply and the demand curves below the autarky equilibrium price $\overline{P}_2^a$. The excess supply and excess demand curves are presented in the trade quadrant of the graph.

Trade equilibrium is reached at the intersection between *ED* and *ES,* where supply price is equal to that of demand and where the market clears. The new equilibrium prices are equal in the two markets and are within the range of the two autarky prices. The quantity trade from Region 1 to Region 2 is equal to the difference between the quantity supplied and demanded in each region. At the new equilibrium, Region 1 produced the quantity $\overline{Q}_1^S$ but consumed only the quantity $\overline{Q}_1^D$, the difference *X* being traded to Region 2, which is producing only quantity $\overline{Q}_2^S$ but consuming the quantity $\overline{Q}_2^D$.

This model allows researchers to assess the gains from trade in the two regions and on the system as a whole. In the Region 1 the consumers lose, but the producers gain more, the net gain being equivalent to the area g_1, which by construction is equal to n_1 in the trade quadrant. In Region 2 the producers lose but the consumers gain more, the net gain being equivalent to the area g_2, which by construction is equivalent to the area n_2. Hence, the net gain for the system would be equivalent to $n_1 + n_2$.

Figure 2 reproduces the same two-market models, but a constant unit transportation cost $T_{1,2}$ is now introduced for trade between Regions 1 and 2. The transport cost shifts the excess supply function upward to *ES[1]*, by the amount of the cost $T_{1,2}$. The new trade equilibrium is reached where the Excess Demand function *ED* crosses the new Excess Supply function *ES[1]*. But at this point, the equilibrium price in Region 2 exceeds the equilibrium price in Region 1 by the level of the transportation cost i.e. $\overline{P}_2 - \overline{P}_1 = T_{1,2}$ or in a more general way, $P_j - P_i \leq T_{ij}$. If the price differential between the two regions is less than the transport cost, trade will not occur. As in the previous case, the net gain for the system is

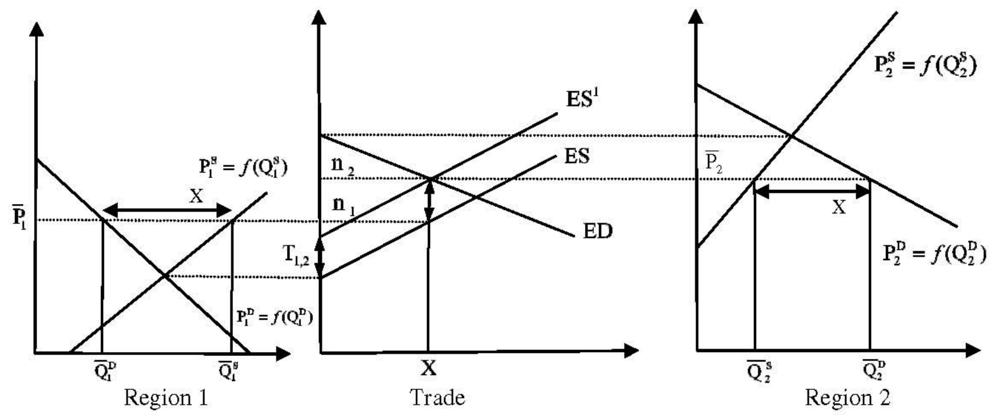

Figure 2. Spatial price equilibrium model with transportation cost.

equivalent to $n_1 + n_2$.

However, the validity of the above analysis seriously relies on the accuracy of the exogenous parameters including supply and demand intercepts and slopes and as well on the transportation cost. Since it is impossible to exactly estimate such data in the empirical research, developing a proper calibration technique is inevitable and can assist researchers and policy makers to make valid decision through the obtained results from SEM.

METHODOLOGY

Based on the graphical framework, STJ developed the following Quadratic Problem (QP) to solve a multi-region SEM:

$$\text{Max} \quad \varphi = \sum_j \int_0^{Q^D} f(Q_j^D)\, dQ_j^D - \sum_i \int_0^{Q^S} f(Q_i^S)\, dQ_i^S - \sum_i \sum_j T_{ij} X_{ij} \qquad (1)$$

s.t.

$$Q_j^D \leq \sum_i X_{ij} \qquad (2)$$

$$Q_i^S \geq \sum_j X_{ij} \qquad (3)$$

$$Q_j^D, Q_i^S, X_{ij} \geq 0 \qquad (4)$$

The objective function which is in a quasi-welfare form, has been termed as net social payoff by Samuelson (1952). The same notation was used in the figures and the models for simplicity. Constraint (2) requires the quantity demanded in each region to be either equal to or smaller than the total volume of in-shipments from throughout all the regions, including itself. Constraint (3) requires the quantity supplied in each region to be equal to or greater than the total volume of out-shipments from that region to all the others, including itself. Therefore the net social payoff is an optimized subject to supply and demand balances and the non-negativity constraint for the optimization variables in each region.

However underlying restrictive assumptions of STJ approach i.e. linear demand and supply functions, whose slope matrices are symmetric and positive semi-definite, not only have limited the application of this framework to multi-commodity analyses but have restricted the usage of such policy instruments as *ad valorem* tariffs and interest rates as well. In the above cases the integrability condition of the system is violated, therefore, it is impossible to conduct a quadratic objective function to be either maximized or minimized. As a remedy, researchers e.g. Minot et al. (1998), Langyintuo et al. (2005), Nolte et al. (2010) and Mosavi et al. (2012-2014) used equilibrium structure with complementarity formulations instead of optimization models. The complementarity models are equal to the first-order KKT conditions of the underlying optimization problems which should be simultaneously solved to find the optimal solution (Devadoss, 2013). In addition, Devadoss (2013) demonstrated that the primal, dual and the complementarity formulation of SEM generate equal solutions. However the complementarity formulation is more efficient in multi-commodity cases with asymmetric slope matrices, common in the real world applications. The STJ model corresponding to Equations (1) to (4) can be rewritten in the complementarity formulations as:

$$P_j^D \geq f(Q_j^D) \qquad \perp Q_j^D \geq 0 \qquad (5)$$

$$f(Q_i^S) \geq P_i^S \qquad \perp Q_i^S \geq 0 \qquad (6)$$

$$Q_j^D \leq \sum_i X_{ij} \qquad \perp P_j^D \geq 0 \qquad (7)$$

$$Q_i^S \geq \sum_j X_{ij} \qquad \perp P_i^S \geq 0 \qquad (8)$$

$$P_j^D - P_i^S \leq T_{ij} \qquad \perp X_{ij} \geq 0 \qquad (9)$$

Where symbol $\perp$ which is named complementarity slackness condition, refers to the orthogonality of each inequality by its complementarity variable and as well non-negativity of that variable. For example in the first inequality $\perp Q_j^D \geq 0$ means $P_j^D - f(Q_j^D)]Q_j^D \geq 0$, and $Q_j^D \geq 0$. The complementarity slackness conditions are equal to zero at the optimal solution and also are the cornerstone of our calibration procedure.

Inequalities (5) and (6) specify inversed demand and supply functions as the function of quantities. Also inequalities (7) and (8) maintain the commodity balance in each region, connecting total quantity demanded and inflow as well as total quantity supplied and outflows respectively. Moreover, inequality (9) refers to the spatial arbitrage in the model. Also, complementary slackness variables provide for prices, quantities and trade flows to become zero if the corresponding inequalities do not hold with strict equality.

Calibrating Multi-commodity SEM Including *Ad valorem* Tariffs

To develop a calibrated model including *ad valorem* tariffs, let $\mathbf{P}_j^D$ and $\mathbf{P}_i^S$ be demand and supply price matrices, $\boldsymbol{\tau}_{ij}$ be matrix of *ad valorem* tariffs, $\mathbf{X}_{ij}$ be the bilateral trade matrix, $\mathbf{T}_{ij}$ be the transportation cost matrix among regions, $\mathbf{A}_j$, $\boldsymbol{\Phi}_j$, $\mathbf{B}_i$ and $\boldsymbol{\Theta}_i$, be demand and supply parameters respectively, and as well $\mathbf{Q}_j^D$ and $\mathbf{Q}_i^S$ be matrices of demand and supply quantities in each region.

Assuming linear demand and supply functions with asymmetric slope matrices one can adopt the following multi-commodity equilibrium structure:

$$\mathbf{P}_j^D \geq \mathbf{A}_j - \boldsymbol{\Phi}_j \mathbf{Q}_j^D \quad \perp \mathbf{Q}_j^D \geq \mathbf{0} \qquad (10)$$

$$\mathbf{P}_i^S \leq \mathbf{B}_i + \boldsymbol{\Theta}_i \mathbf{Q}_i^S \quad \perp \mathbf{Q}_i^S \geq \mathbf{0} \qquad (11)$$

$$\sum\nolimits_i \mathbf{X}_{ij} \geq \mathbf{Q}_j^D \quad \perp \mathbf{P}_j^D \geq \mathbf{0} \qquad (12)$$

$$\sum\nolimits_j \mathbf{X}_{ij} \leq \mathbf{Q}_i^S \quad \perp \mathbf{P}_i^S \geq \mathbf{0} \qquad (13)$$

$$\left[\mathbf{P}_i^S + \mathbf{T}_{ij}\right](1+\boldsymbol{\tau}_{ij}) \geq \mathbf{P}_j^D \quad \perp \mathbf{X}_{ij} \geq \mathbf{0} \qquad (14)$$

Inequality (14) is similar to inequality (9) with the same interpretation, but only adjusted by *ad valorem* tariffs. Now let $\boldsymbol{\alpha}_j$, $\boldsymbol{\varphi}_j$, $\boldsymbol{\beta}_i$ and $\boldsymbol{\theta}_i$ be deviations from demand and supply parameters respectively, $\overline{\mathbf{X}}_{ij}$ be realized trade matrix and $\boldsymbol{\Gamma}_{ij}$ the implicit trade cost among regions, e.g. the effects of nontariff barriers. Following Paris *et al.* (2009) procedures, a least-squares approach, subject to the equilibrium structure, (10) to (14) and calibrated constraint (18) was used to estimate all the above parameters at the stage I.

The first four constituents of the objective function within the first bracket are the sum of squared deviations of demand and supply parameters and while the remaining components are complementary slackness conditions of the equilibrium structure (10-14) which are equal to zero at the optimal solution.

$$\begin{aligned}\min_{\boldsymbol{\alpha}_i,\boldsymbol{\beta}_i,\boldsymbol{\varphi}_i,\boldsymbol{\theta}_i,\boldsymbol{\Gamma}_{ij}} \quad \Omega = &\frac{1}{2}\left[\sum\nolimits_j \boldsymbol{\alpha}_j'\boldsymbol{\alpha}_j + \sum\nolimits_i \boldsymbol{\beta}_i'\boldsymbol{\beta}_i + \sum\nolimits_j \mathrm{trace}(\boldsymbol{\varphi}_j'\boldsymbol{\varphi}_j) + \sum\nolimits_i \mathrm{trace}(\boldsymbol{\theta}_i'\boldsymbol{\theta}_i)\right] \\ &-(\sum\nolimits_j [\mathbf{A}_j + \boldsymbol{\alpha}_j] - [\boldsymbol{\Phi}_j + \boldsymbol{\varphi}_j]\,\mathbf{Q}_j^D)'\,\mathbf{Q}_j^D + \sum\nolimits_i \sum\nolimits_j \mathbf{P}_i^S \boldsymbol{\tau}_{ij} \mathbf{X}_{ij} \\ &+(\sum\nolimits_i [\mathbf{B}_i + \boldsymbol{\beta}_i] + [\boldsymbol{\Theta}_i + \boldsymbol{\theta}_i]\mathbf{Q}_i^S)'\,\mathbf{Q}_i^S \qquad (15) \\ &+\sum\nolimits_i \sum\nolimits_j [\overline{\mathbf{X}}_{ij}' \boldsymbol{\Gamma}_{ij}(1+\boldsymbol{\tau}_{ij}) + \mathbf{T}_{ij}'(1+\boldsymbol{\tau}_{ij})\mathbf{X}_{ij}]\end{aligned}$$

s.t.

$$\sum\nolimits_i \mathbf{X}_{ij} \geq \mathbf{Q}_j^D \qquad (16)$$

$$\sum\nolimits_j \mathbf{X}_{ij} \leq \mathbf{Q}_i^S \qquad (17)$$

$$\mathbf{X}_{ij} = \overline{\mathbf{X}}_{ij} \qquad (18)$$

$$\mathbf{P}_i^S + [\mathbf{T}_{ij} + \boldsymbol{\Gamma}_{ij}] \geq \mathbf{P}_j^D[1/(1+\boldsymbol{\tau}_{ij})] \qquad (19)$$

$$\mathbf{P}_j^D \geq [\mathbf{A}_j + \boldsymbol{\alpha}_j] - [\boldsymbol{\Phi}_j + \boldsymbol{\varphi}_j]\mathbf{Q}_j^D \qquad (20)$$

$$\mathbf{P}_i^S \leq [\mathbf{B}_i + \boldsymbol{\beta}_i] + [\boldsymbol{\Theta}_i + \boldsymbol{\theta}_i]\mathbf{Q}_i^S \qquad (21)$$

$$\mathbf{Q}_j^D, \mathbf{Q}_i^S, \mathbf{X}_{ij}, \mathbf{P}_i^S, \mathbf{P}_j^D \geq \mathbf{0} \qquad (22)$$

At stage II, the estimated parameters from stage I, i.e. $\hat{\boldsymbol{\alpha}}_j$, $\hat{\boldsymbol{\varphi}}_j$, $\hat{\boldsymbol{\beta}}_i$, $\hat{\boldsymbol{\theta}}_i$ and $\hat{\boldsymbol{\Gamma}}_{ij}$ are included in the equilibrium structure to calibrate the observations of the base year. Let $\boldsymbol{\varepsilon}_{ij}^{X}, \boldsymbol{\varepsilon}_{j}^{Q^D}, \boldsymbol{\varepsilon}_{i}^{Q^S}, \boldsymbol{\varepsilon}_{i}^{P^S}$ and $\boldsymbol{\varepsilon}_{j}^{P^D}$ be positive slack variables corresponding to $\mathbf{X}_{ij}, \mathbf{Q}_j^D, \mathbf{Q}_i^S, \mathbf{P}_i^S$ and $\mathbf{P}_j^D$ respectively. Restructuring the model in a minimization form by using an auxiliary objective function yields:

$$Min \sum_j \boldsymbol{\varepsilon}_j^{Q^D} \mathbf{P}_j^D + \sum_i \boldsymbol{\varepsilon}_i^{Q^S} \mathbf{P}_i^S + \sum_j \boldsymbol{\varepsilon}_j^{P^D} \mathbf{Q}_j^D + \sum_i \boldsymbol{\varepsilon}_i^{P^S} \mathbf{Q}_i^S + \sum_i \sum_j \boldsymbol{\varepsilon}_{ij}^{X} \mathbf{X}_{ij} \quad (23)$$

s.t.

$$\sum_i \mathbf{X}_{ij} = \mathbf{Q}_j^D + \boldsymbol{\varepsilon}_j^{Q^D} \quad (24)$$

$$\sum_j \mathbf{X}_{ij} + \boldsymbol{\varepsilon}_i^{Q^S} = \mathbf{Q}_i^S \quad (25)$$

$$\mathbf{P}_i^S + [\mathbf{T}_{ij} + \hat{\boldsymbol{\Gamma}}_{ij}] = \mathbf{P}_j^D [1/(1+\boldsymbol{\tau}_{ij})] + \boldsymbol{\varepsilon}_{ij}^X \quad (26)$$

$$\mathbf{P}_j^D = [\mathbf{A}_j + \hat{\boldsymbol{\alpha}}_j] - [\boldsymbol{\Phi}_j + \hat{\boldsymbol{\varphi}}_j]\mathbf{Q}_j^D + \boldsymbol{\varepsilon}_j^{P^D} \quad (27)$$

$$[\mathbf{B}_i + \hat{\boldsymbol{\beta}}_i] + [\boldsymbol{\Theta}_i + \hat{\boldsymbol{\theta}}_i]\mathbf{Q}_i^S = \mathbf{P}_i^S + \boldsymbol{\varepsilon}_i^{P^S} \quad (28)$$

$$\mathbf{Q}_j^D, \mathbf{Q}_i^S, \mathbf{X}_{ij}, \mathbf{P}_i^S, \mathbf{P}_j^D, \boldsymbol{\varepsilon}_{ij}^X, \boldsymbol{\varepsilon}_j^{Q^D}, \boldsymbol{\varepsilon}_i^{Q^S}, \boldsymbol{\varepsilon}_i^{P^S}, \boldsymbol{\varepsilon}_j^{P^D} \geq \mathbf{0} \quad (29)$$

Since the auxiliary objective, i.e. Equation (23) is the sum of all complementary slackness conditions, it is equal to zero at the optimal solution. By doing either simple mathematical operation or by computing KKT conditions of the above minimization model, it becomes evident that Equation (23) satisfies the equilibrium structure. All other equations are the same as the equilibrium structure (10) to (14) with the same interpretations, only adjusted by deviations and slacks. The minimization structure (23) to (29) which encloses *ad valorem* tariffs can perfectly calibrate the observed information of the base year.

Generalization to a Dynamic Model

Generalizing the above model to a temporal one needs the addition of a sequence condition to the initial equilibrium structure (10) to (14). The sequence condition links the SEM during time periods by introducing storage activity in the model. In the complementarity formulation, storage activity appears in the model as the complementarity slack variables corresponding to sequence condition. Therefore if $t = 1,...,n,...,N$ refers to time periods, the equilibrium structure (10) to (14) can be altered as follows:

$$\mathbf{P}_{tj}^D \geq \mathbf{A}_j - \boldsymbol{\Phi}_j \mathbf{Q}_{tj}^D \quad \perp \mathbf{Q}_{tj}^D \geq \mathbf{0} \quad (30)$$

$$\mathbf{P}_{ti}^S \leq \mathbf{B}_i + \boldsymbol{\Theta}_i \mathbf{Q}_{ti}^S \quad \perp \mathbf{Q}_{ti}^S \geq \mathbf{0} \quad (31)$$

$$\sum_i \mathbf{X}_{tij} \geq \mathbf{Q}_{tj}^D + \mathbf{S}_{(t+n)j} \quad \perp \mathbf{P}_{tj}^D \geq \mathbf{0} \quad (32)$$

$$\sum_j \mathbf{X}_{tij} \leq \mathbf{Q}_{ti}^S + \mathbf{S}_{ti} \quad \perp \mathbf{P}_{ti}^S \geq \mathbf{0} \quad (33)$$

$$[\mathbf{P}_{ti}^S + \mathbf{T}_{ij}]\,(1+\boldsymbol{\tau}_{ij}) \geq \mathbf{P}_{tj}^D \quad \perp \mathbf{X}_{tij} \geq \mathbf{0} \quad (34)$$

$$[\mathbf{P}_{(t-n)i}^S + \boldsymbol{\Sigma}_i]\,(1+\boldsymbol{\gamma}_i)^n \geq \mathbf{P}_{ti}^D \quad \perp \mathbf{S}_{ti} \geq \mathbf{0} \quad (35)$$

Where, $\boldsymbol{\Sigma}_i$ refers to storage costs matrix, $\mathbf{S}_{ti}$ is the matrix of storage quantities and $\boldsymbol{\gamma}_i$ is the interest rate vector in each region. Inequality (35) introduces the time sequence condition in the model. The subscript n refers to time lag. For instance n is equal to one in the case of storing between two consecutive periods. If the price differential between the two time periods is less than the storage cost, storage activity will not occur. In other words, if the future value of stored commodities becomes less than the present value of the same commodities, storage activity will not occur. In this regard, interest rate plays an important role because it can directly influence the future value of the current values. In the presence of storage, the commodity balance condition should be modified. Now inflows should satisfy demand and storage quantity for the future periods (inequality 32) and current supply and storage quantities from last periods should fulfill the market outflow (inequality 33).

Therefore the new SEM corresponds to inequalities (30) to (35), including both *ad valorem* tariffs and interest rates, being suitable to model more complicated real world's problems.

Assuming storage costs are also imprecisely measured, the least-squares model can be modified as:

$$
\begin{aligned}
\underset{\boldsymbol{\alpha}_{ti},\,\boldsymbol{\beta}_{ti},\boldsymbol{\varphi}_{ti},\,\boldsymbol{\theta}_{ti},\boldsymbol{\Gamma}_{tij},\boldsymbol{\delta}_{ti}}{\text{Min}} \quad \Omega = \sum\nolimits_t (1+\boldsymbol{\gamma}_i)^{-t}\Bigg[& \frac{1}{2}[\sum\nolimits_j \boldsymbol{\alpha}'_{tj}\boldsymbol{\alpha}_{tj} + \sum\nolimits_i \boldsymbol{\beta}'_{ti}\boldsymbol{\beta}_{ti} + \sum\nolimits_j \text{trace}(\boldsymbol{\varphi}'_{tj}\boldsymbol{\varphi}_{tj}) + \sum\nolimits_i \text{trace}(\boldsymbol{\theta}'_{ti}\boldsymbol{\theta}_{ti})] \\
& -(\sum\nolimits_j [\mathbf{A}_j + \boldsymbol{\alpha}_{tj}] - [\boldsymbol{\Phi}_j + \boldsymbol{\varphi}_{tj}]\,\mathbf{Q}^D_{tj})'\,\mathbf{Q}^D_{tj} - \sum\nolimits_i \mathbf{S}'_{(t+n)j}\mathbf{P}^D_{tj} \\
& +(\sum\nolimits_i [\mathbf{B}_i + \boldsymbol{\beta}_{ti}] + [\boldsymbol{\Theta}_i + \boldsymbol{\theta}_{ti}]\mathbf{Q}^S_{ti})'\,\mathbf{Q}^S_{ti} - \sum\nolimits_i \mathbf{S}'_{ti}(\mathbf{P}^D_{ti} - \mathbf{P}^S_{ti}) \\
& +\sum\nolimits_i [(\mathbf{P}^S_{(t-n)i} + \boldsymbol{\delta}'_{ti})(1+\boldsymbol{\gamma}_i)^n \bar{\mathbf{S}}_{ti} + \boldsymbol{\Sigma}'_i(1+\boldsymbol{\gamma}_i)^n \mathbf{S}_{ti}] \\
& +\sum\nolimits_i \sum\nolimits_j (\mathbf{P}^S_{ti}\boldsymbol{\tau}_{ij}\mathbf{X}_{tij} + \bar{\mathbf{X}}'_{tij}\boldsymbol{\Gamma}_{tij}(1+\boldsymbol{\tau}_{ij}) + \mathbf{T}'_{ij}(1+\boldsymbol{\tau}_{tij})\mathbf{X}_{tij})\Bigg]
\end{aligned} \tag{36}
$$

s.t.

$$\sum\nolimits_i \mathbf{X}_{tij} \geq \mathbf{Q}^D_{tj} + \mathbf{S}_{(t+n)j} \tag{37}$$

$$\sum\nolimits_j \mathbf{X}_{tij} \leq \mathbf{Q}^S_{ti} + \mathbf{S}_{ti} \tag{38}$$

$$\mathbf{X}_{tij} = \bar{\mathbf{X}}_{tij} \tag{39}$$

$$\mathbf{S}_{ti} = \bar{\mathbf{S}}_{ti} \tag{40}$$

$$\mathbf{P}^S_{ti} + [\mathbf{T}_{ij} + \boldsymbol{\Gamma}_{tij}] \geq \mathbf{P}^D_{tj}[1/(1+\boldsymbol{\tau}_{ij})] \tag{41}$$

$$[\mathbf{P}^S_{(t-n)i} + (\boldsymbol{\Sigma}_i + \boldsymbol{\delta}_{ti})]\ (1+\boldsymbol{\gamma}_i)^n \geq \mathbf{P}^D_{ti} \tag{42}$$

$$\mathbf{P}^D_{tj} \geq [\mathbf{A}_j + \boldsymbol{\alpha}_{tj}] - [\boldsymbol{\Phi}_j + \boldsymbol{\varphi}_{tj}]\mathbf{Q}^D_{tj} \tag{43}$$

$$\mathbf{P}^S_{ti} \leq [\mathbf{B}_i + \boldsymbol{\beta}_{ti}] + [\boldsymbol{\Theta}_i + \boldsymbol{\theta}_{ti}]\mathbf{Q}^S_{ti} \tag{44}$$

$$\mathbf{Q}^D_{tj}, \mathbf{Q}^S_{ti}, \mathbf{X}_{tij}, \mathbf{S}_{ti}, \mathbf{P}^S_{ti}, \mathbf{P}^D_{tj} \geq \mathbf{0} \tag{45}$$

Where, $\boldsymbol{\delta}_{ti}$ stands for such implicit storage costs as opportunity costs and $\bar{\mathbf{S}}_{ti}$ denoting the realized storage quantities in the base year. The objective function (Equation 36) minimizes the sum of squared deviations of demand and supply parameters and the discounted value of all the complementarity slackness conditions subject to equilibrium structure (30) to (35) and calibrated constraints (39) and (40). In this new least-squares model, four components related to complementary slackness conditions of the sequence condition were included. In addition to other parameters, this model is qualified enough to estimate the implicit storage costs which are used to make up for observed storage costs. Comprising $\boldsymbol{\varepsilon}^S_{ti}$ as positive slack variable corresponds to $\mathbf{S}_{ti}$, the stage II of the calibration procedure would perform as:

Again the auxiliary objective function which is the sum of the discounted value of

$$\text{Min} \quad \sum\nolimits_t (1+\boldsymbol{\gamma}_i)^{-t}[\sum\nolimits_j \boldsymbol{\varepsilon}^{Q^D}_{tj}\mathbf{P}^D_{tj} + \sum\nolimits_i \boldsymbol{\varepsilon}^{Q^S}_{ti}\mathbf{P}^S_{ti} + \sum\nolimits_j \boldsymbol{\varepsilon}^{P^D}_{tj}\mathbf{Q}^D_{tj} + \sum\nolimits_i \boldsymbol{\varepsilon}^{P^S}_{ti}\mathbf{Q}^S_{ti} + \sum\nolimits_i \sum\nolimits_j \boldsymbol{\varepsilon}^X_{tij}\mathbf{X}_{tij} + \sum\nolimits_i \boldsymbol{\varepsilon}^S_{ti}\mathbf{S}_{ti}] \tag{46}$$

s.t.

$$\sum\nolimits_i \mathbf{X}_{tij} = \mathbf{Q}^D_{tj} + \mathbf{S}_{(t+n)j} + \boldsymbol{\varepsilon}^{Q^D}_{tj} \tag{47}$$

$$\sum\nolimits_j \mathbf{X}_{tij} + \boldsymbol{\varepsilon}^{Q^S}_{ti} = \mathbf{Q}^S_{ti} + \mathbf{S}_{ti} \tag{48}$$

$$\mathbf{P}^S_{ti} + [\mathbf{T}_{ij} + \hat{\boldsymbol{\Gamma}}_{tij}] = \mathbf{P}^D_{tj}[1/(1+\boldsymbol{\tau}_{tij})] + \boldsymbol{\varepsilon}^X_{tij} \tag{49}$$

$$[\mathbf{P}^S_{(t-n)i} + (\boldsymbol{\Sigma}_i + \hat{\boldsymbol{\delta}}_{ti})](1+\boldsymbol{\gamma}_i)^n = \mathbf{P}^D_{ti} + \boldsymbol{\varepsilon}^S_{ti} \tag{50}$$

$$\mathbf{P}^D_{tj} = [\mathbf{A}_j + \hat{\boldsymbol{\alpha}}_{tj}] - [\boldsymbol{\Phi}_j + \hat{\boldsymbol{\varphi}}_{tj}]\mathbf{Q}^D_{tj} + \boldsymbol{\varepsilon}^{P^D}_{tj} \tag{51}$$

$$[\mathbf{B}_i + \hat{\boldsymbol{\beta}}_{ti}] + [\boldsymbol{\Theta}_i + \hat{\boldsymbol{\theta}}_{ti}]\mathbf{Q}^S_{ti} = \mathbf{P}^S_{ti} + \boldsymbol{\varepsilon}^{P^S}_{ti} \tag{52}$$

$$\mathbf{Q}^D_{tj}, \mathbf{Q}^S_{ti}, \mathbf{X}_{tij}, \mathbf{P}^S_{ti}, \mathbf{P}^D_{tj}, \mathbf{S}_{ti}, \boldsymbol{\varepsilon}^X_{tij}, \boldsymbol{\varepsilon}^{Q^D}_{tj}, \boldsymbol{\varepsilon}^{Q^S}_{ti}, \boldsymbol{\varepsilon}^{P^S}_{ti}, \boldsymbol{\varepsilon}^{P^D}_{tj}, \boldsymbol{\varepsilon}^S_{ti} \geq \mathbf{0} \tag{53}$$

all complementary slackness conditions, is equal to zero at the optimal solution. Therefore Equations (46) to (53) are the adjusted versions of the equilibrium structure (30) to (35) with equivalent explanation. These sets of equations which enclose *ad valorem* tariffs, construct a spatial and temporal SEM that can exactly reproduce all the observed information.

An Illustrative Example

Considering the previous notation, a simple 2×2×2 spatial and temporal trade model is presented to justify model performance whose initial data have been obtained from Mosavi (2012). In this example, two regions (R1 and R2) trade two commodities (Rice and Maize) in two time periods. The example is programmed in General Algebraic Modeling System (GAMS) so that all observed prices and quantities are used as initial values in the search by the solver of the equilibrium solution.

Table 1 demonstrates the initial values (IV), correction terms (COR) and the adjusted values (ADJ) for demand and supply function's intercepts as well as storage cost. The correction terms are gathered from stage I of the calibration procedure. In this example, the correction terms for intercepts were estimated too small to adjust for the initial value of intercepts. However the least-squares model estimates the non-zero correction term to adjust for the storage costs which are assumed as imprecisely measured.

Similar to Table 1, Table 2 presents the slopes of demand and supply functions. Again IV, COR and ADJ refer to initial values, correction terms and the adjusted values. Like intercepts, the correction terms

Table 1. Demand and supply intercepts and storage costs.

	R1						R2					
	Rice			Maize			Rice			Maize		
	IV [a]	COR [b]	ADJ [c]	IV	COR	ADJ	IV	COR	ADJ	IV	COR	ADJ
Demand intercepts	30	0	30	55	0	55	22	0	22	28	0	28
Supply intercepts	0.4	0	0.4	0.1	0	0.1	0.2	0	0.2	-0.4	0	-0.4
Storage cost	6	11.04	17.04	5.5	24.59	30.09	7	-3.17	3.38	3.2	8.10	11.3

[a] Initial values, [b] Correction terms and the [c] Adjusted values respectively.

Table 2. Demand and supply slopes.

Demand slope												
	R1						R2					
	Rice			Maize			Rice			Maize		
	IV [a]	COR [b]	ADJ [c]	IV	COR	ADJ	IV	COR	ADJ	IV	COR	ADJ
Rice	-0.5	0	-0.5	-0.2	0	-0.2	-0.5	0	-0.5	-0.4	0	-0.4
Maize	0.3	0	0.3	-2.1	0	-2.1	0.2	0	0.2	-1.0	0	-1.0
Supply slope												
	R1						R2					
	Rice			Maize			Rice			Maize		
	IV	COR	ADJ	IV	COR	ADJ	IV	COR	ADJ	IV	COR	ADJ
Rice	1.4	0	1.4	-0.3	0	-0.3	2.4	0	2.4	0.5	0	0.5
Maize	-0.2	0	-0.2	2.4	0	2.4	0.7	0	0.7	1.6	0	1.6

[a] Initial values, [b] Correction terms and the [c] Adjusted values respectively

Table 3. Transportation cost of commodities among regions.

	Rice						Maize					
	R1			R2			R1			R2		
	IV [a]	COR [b]	ADJ [c]	IV	COR	ADJ	IV	COR	ADJ	IV	COR	ADJ
R1	0	4.4	4.4	4.3	-15.5	-11.2	0	31.9	31.9	6.5	-1.07	5.43
R2	14.5	-12.78	1.72	0	-11.7	11.7	9.5	13.35	22.85	0	4.4	4.4

[a] Initial values, [b] Correction terms and the [c] Adjusted values respectively

for the slopes were estimated too small, hence the IV column and ADJ column were considered equal in Table 2. Likewise, Table 3 presents transportation costs of commodities among regions. As expected, non-zero correction terms were calculated in the least-squares model to make up for the ill- measured transportation costs. The negative value for the adjusted transportation costs refers to the effects of such missing policy instruments as export subsidies. When export subsidies exceed the sum of other transaction costs, the total effective transaction cost among regions may become negative, as in this numerical example.

Other required data are interest rates and *ad valorem* tariffs. The interest rates are considered as equal to 15 percent in the two regions. Also, the *ad valorem* tariffs are shown in Table 4 as a percentage of transportation costs between the two regions. For example, Maize trade between regions 1 and 2 is accompanied by 60 percent extra transportation cost.

The model generates solutions for the endogenous variables within the two time periods, however only the solutions for the first time period were shown in Tables 5 and 6 for the sake of brevity. In these two tables,

Table 4. *Ad valorem* tariffs.

	Rice		Maize	
	R1	R2	R1	R2
R1	0.0	0.1	0.0	0.6
R2	0.1	0.0	0.1	0.0

Table 5. Model solution for demand, supply and Storage quantities and Supply Price.

	R1						R2					
	Rice			Maize			Rice			Maize		
	IV [a]	MS [d]	DIF [e]	IV	MS	DIF	IV	MS	DIF	IV	MS	DIF
Demand quantity	16	16	0	12	12	0	20	20	0	19	19	0
Supply quantity	11	11	0	2	2	0	6	6	0	3	3	0
Supply price	15.2	15.2	0	2.7	2.7	0	16.1	16.1	0	8.6	8.6	0
Storage quantity	7	7	0	11	11	0	12	12	0	15	15	0

Table 6. Model solution for bilateral trade matrix.

	Rice						Maize					
	R1			R2			R1			R2		
	IV	MS	DIF	IV	MS	DIF	IV	MS	DIF	IV	MS	DIF
R1	13	13	0	5	5	0	6	6	0	7	7	0
R2	3	3	0	15	15	0	6	6	0	12	12	0

In Tables 5 and 6, [a] Initial values, [b] Correction terms, [c] Adjusted values respectively. [d] Model solution, [e] difference between IV and MS.

MS refers to the model solution while DIF standing for the difference between IV and MS, showing the calibration error.

Results finally indicate that the proposed spatial and temporal trade model can exactly reproduce base year quantities and prices. This simple model can be easily modified to fit more complicated actual cases, because the results are given by the mathematical formulas which were formerly explained.

Eventually, to show the importance of the results, one can compare the welfare measures obtained from the calibrated model (Equations 46 to 53) and un-calibrated model (Equations 30 to 35). It is important to note that the present models cannot directly compute such welfare measures as consumers' and producers' surplus, because the objective functions of the underlying STJ approach for our equilibrium structures are not welfare functions. In fact, it is net social payoff in primal and is negative social cost in dual modeling approach. As a result, additional computations are needed to estimate the consumer and producer surplus values after having the models solved. Table 7 reveals the present values of consumers and producers surplus as well as tariff revenue for each region.

Also, net welfare is shown in Table 7 as the sum of discounted value of regional consumers and producers surplus as well as tariff revenue. In this table CALIB and UNCALIB refer to calibrated and un-calibrated models respectively. Results show that there is a wide gap among welfare measures resulted from calibrated *vs.* un-calibrated models. This gap may distort any welfare analysis based on un-calibrated models. This is a fact not considered in the SEM literature.

CONCLUSIONS

A positive spatial and temporal trade model was developed to compensate for the poor quality data and for the ill-measured parameters, to enhance the capability of such models to be used in counterfactual simulations. The famous trade tools i.e. Interest rate and *ad valorem* tariffs were adopted in a structural equilibrium model. Next, by attaching linear cost terms to trade and storage flows, it could truly replicate all the observed prices and quantities of the base year. Also the results demonstrated that a wide discrepancy exists among welfare measures from calibrated *vs.* un-calibrated models. Therefore it is strongly suggested that any welfare analysis be conducted through the calibrated model.

Finally, further remarks on the wider extension and implication of the calibrated model can be offered. For example, not only it would be easy to use nonlinear supply and demand functions, but also it becomes possible to consider income as an endogenous variable within the model. Furthermore, the calibrated model is qualified enough to take into account all other such trade policy instruments as exchange rate, trade quotas, and tariff rate quota as well. In these cases, not considered in this article, other complementary slackness conditions have to be added to the model. Moreover, the suggested model is an inter-country (region) one and further

Table 7. Welfare measures from calibrated *vs.* un-calibrated model.

	Consumers Surplus		Producers Surplus		Tariff Revenue		Net Welfare	
	CALIB [a]	UN-CALIB [b]	CALIB	UN-CALIB	CALIB	UN-CALIB	CALIB	UN-CALIB
R1	34694.92	46859.41	-34233.76	-42168.35	8.68	0	469.85	4691.06
R2	651.73	1024.62	-561.63	-685.83	1.66	0	91.76	338.79

[a] Calibrated , [b] Un-calibrated model respectively.

research efforts can be directed towards calibrating intra-country model, with endogenous import and export quantities.

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مدل کالیبره تحلیل تجارت محصولات کشاورزی و غذا همراه با تعرفه‌ی ارزشی

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چکیده

نرخ بهره و تعرفه‌های ارزشی از مهم ترین ابزار تحلیل تجارت بشمار می‌روند که به صورت گسترده در چهارچوب مدل‌ای تعادل فضایی جهت تجزیه و تحلیل تجارت محصولات کشاورزی و غذا در سر تا سر دنیا مورد اتفاده قرار می‌گیرند. با این حال نتایج حاصل از مدل‌ها همواره بحث برانگیز بوده است چرا که این مدل ها نمی‌توانند به خوبی مشاهدات اولیه را باز تولید نمایند. از این رو در مطالعه‌ی حاضر یک مدل کالیبره تحلیل تجارت که بتواند دو ابزار سیاستی نرخ بهره و تعرفه‌ی ارزشی را مورد بررسی قرار دهد، بسط و توسعه یافت. با استفاده از این مدل کالیبره محققین می‌توانند مدلی سالم و بی عیب و نقص جهت تحلیل‌های تجاری در بخش کشاورزی تهیه نمایند. در پایان یک مثال عددی جهت اثبات صحت مدل و نیز جهت مقایسه‌ی تحلیل رفاه در دو مدل کالیبره (مدل جدید) شده و کالیبره (مدل‌های مرسوم) نشده ارائه گردیده است.