

## Estimation of Unsaturated Soil Hydrodynamic Parameters Using Inverse Problem Technique

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### ABSTRACT

Mathematical simulation of flow toward drains is an important and indispensable stage in drainage design and management. Many related models have been developed, but most of them simulate the saturated flow toward drains without a due consideration of the unsaturated zone. In this study, the two dimensional differential equation governing saturated and unsaturated flow in porous media is numerically solved and water table variations between drains predicted. By introducing and linking a proper optimization model to the numerical one, saturated and unsaturated soil hydrodynamic parameters were estimated within the inverse problem technique context. Data for calibration and verification were provided through a conduction of laboratory experimentation. Other laboratory data were also employed for the proposed model evaluation. The results indicated that in addition to a prediction of the water table variations between drains, the inverse problem model can be employed to estimate the unsaturated soil hydrodynamic parameters with a high degree of precision.

**Keywords:** Numerical model, Inverse problem technique, Saturated flow, Unsaturated flow.

### INTRODUCTION

The unsaturated root-zone soil plays a significant role in moisture flow, salt transfer as well as in the growth of plants (Ryan, 2008). A drainage system extends the unsaturated zone through a drawdown of the water table. Due to the gravity part of the unsaturated zone, water flows toward drains as excess water.

Mathematical modeling of flow in saturated-unsaturated soil zone is an important tool through which soil moisture and water table variations between drains are predicted. Richard's equation is a partial differential equation describing soil water dynamics in modeling saturated-unsaturated flow in porous media (Clement *et al.*, 1994). Due to the nonlinearity of the equation, it

cannot be solved analytically and has to be solved by employing numerical methods (Simpson and Clement, 2003; Ramos *et al.*, 2006).

Simpson and Clement (2003) compared the application of finite difference and finite element methods as applied to porous media flows and demonstrated that the finite element methods are more precise than the finite difference ones. Whilliam *et al.* (1994) simulated saturated-unsaturated flow toward open drains using the finite difference method. They concluded that water table profile and seepage zone length along drain open sides are functions of capillary force in unsaturated zone. Fipps *et al.* (1986) solved partial differential equation of saturated-unsaturated flow numerically, using the finite element method. They showed that through a use of suitable boundary

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conditions, this method can precisely predict the water table depth near the drains. Other applications of finite difference and finite element methods for saturated-unsaturated flow in porous media can be found in literature (Clement *et al.*, 1994; Gureghian and Youngs, 1975; Nature *et al.*, 1975; Yeh and King, 1978; Fausey, 1975; Schwaerzel *et al.*, 2006; Vrugt *et al.*, 2001).

One of the serious problems encountered in drainage models is a determination of input parameters. This problem can restrict model applications for design and management purposes. Saturated and unsaturated soil hydraulic properties are the most important input parameters that vary spatially as well as temporally. Therefore, assigning values to these parameters is accompanied with error and uncertainty, making it necessary to find a new way to overcome the problem (Van Genuchten and Leij, 1992; Warrick and Myers, 1987; Moustafa, 2000; Vrugt *et al.*, 2001).

A new method of field measurement of soil parameters and their interpretation is an inverse problem technique. In this method, difficult direct measurements are achieved by measuring the easily available variables (Ritter *et al.*, 2003). This technique has many applications in engineering and such sciences as hydraulics and soil physics. Dane and Hruska (1983) used inverse problem technique to estimate soil moisture characteristic curve in homogeneous soils. Other researchers extended and improved the use of inverse problem technique in the field (Ritter *et al.*, 2003; Olyphant, 2003; Jhorar *et al.*, 2002; Abbaspour *et al.*, 1999; Stefan and Boris, 1999; Nutzman *et al.*, 1998; Simunek *et al.*, 1998; Kumar *et al.*, 1994; Bitterlich *et al.*, 2004).

In this research, the finite volume numerical method is employed to solve Richard's equation numerically for simulation of unsteady saturated-unsaturated flow toward drain conduits. Also, by employing the measured water table variations between drains, which is relatively easy as compared with direct measurements of unsaturated soil properties,

and the numerical model within the inverse problem technique context, average and effective values of saturated and unsaturated soil hydraulic properties can be estimated.

## METHODOLOGY

### Model Development

In the present research, the governing partial differential equation of saturated-unsaturated flow toward drains was solved numerically using the finite volume method and this model was then incorporated within the inverse problem technique. In numerical models, as compared to analytical ones, much more complicated geometry and boundary conditions can be analyzed.

### Solution Domain

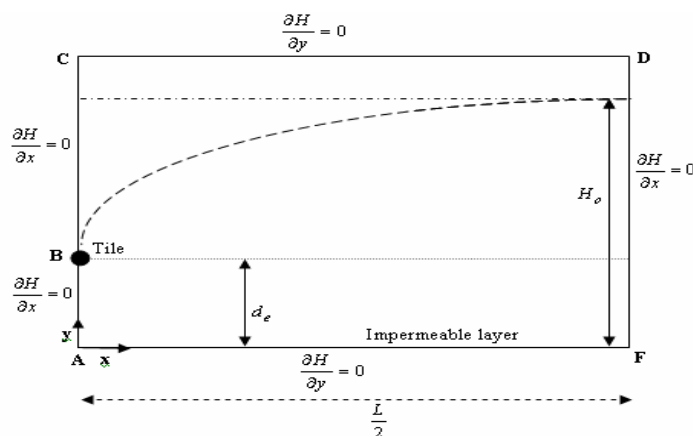
Figure 1 shows a vertical section, symbols and boundary conditions of a horizontal drainage system (point B is a drain). In this Figure,  $L$  is the distance between drains,  $H$  the water table height over an impermeable layer,  $d_e$  is the vertical distance between the drain and impermeable layer and  $H_0$  an initial water table height over the impermeable layer.

### Flow Equation

Richard's equation for a saturated-unsaturated 2-D flow in a heterogeneous and anisotropic soil is given as:

$$\frac{\partial}{\partial x} \left[ K_x \frac{\partial H}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_y \frac{\partial H}{\partial y} \right] = \frac{\partial \theta}{\partial t} \quad (1)$$

where  $H$  is the hydraulic head equal to  $Z + h_p$ ;  $K_y$  and  $K_x$  are the vertical and horizontal unsaturated hydraulic conductivity for unsaturated condition respectively;  $\theta$  is the volumetric soil moisture and  $Z$  the vertical distance to impermeable layer, being called gravitational potential;  $h_p$  is the



**Figure 1.** Schematic, symbols and boundary conditions representation of 2D unsteady-state drainage.

pressure head, while  $K_y, K_x$  and  $\theta$  the functions of pressure head ( $h_p$ ).

### Soil Hydrodynamic Properties Function

In this study, the soil hydrodynamic properties function and soil moisture characteristic curve proposed by Brooks and Corey (1964) were employed.

a) Soil moisture characteristic curve

$$\theta = \theta_r + (\theta_s - \theta_r) \left(\frac{h_b}{h}\right)^\lambda \text{ for } h < h_b \quad (2)$$

$$\theta = \theta_s \text{ for } h \geq h_b \quad \eta = +3\lambda$$

where  $h$  is the pressure head,  $\theta$  is the soil moisture,  $\theta_s$  is the saturated soil moisture,  $\theta_r$  is the residual soil moisture,  $h_b$  is the pressure potential at the time of air entry and  $\lambda$  is the empirical soil parameter.

b) Unsaturated hydraulic conductivity function

$$K = K_s \left[\frac{h_b}{h}\right]^\eta \text{ for } h < h_b \quad (3)$$

$$K = K_s \text{ for } h \geq h_b$$

where  $K$  is the unsaturated hydraulic conductivity,  $K_s$  is the saturated hydraulic conductivity and  $\eta$  is the empirical constant which is calculated from  $\eta = 2 + 3\lambda$ .

### Initial and Boundary Conditions

In Figure 1, initial and boundary conditions can be defined as:

a) Initial conditions

$$0 \leq x \leq \frac{L}{2}$$

$$H(x, y, t) = H_0, \quad 0 \leq y \leq y_D \quad (4a)$$

$$t = 0$$

where  $H_0$  is the initial water table above impermeable layer and given as constant;  $y_D$  is the vertical distance between point D and the impermeable layer; and  $H(x, y, t)$  is the hydraulic head of point  $(x, y)$  at time  $t$ .

b) Boundary conditions

Boundary conditions are shown in Figure 1 and are defined as follows:

$$H(x, y, t) = d_e, \quad x = x_B, y = y_B, t > 0$$

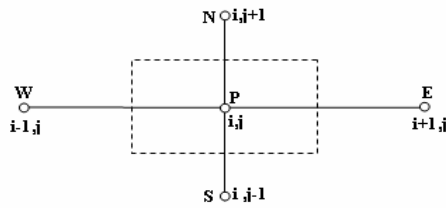
$$\frac{\partial H(x, y, t)}{\partial x} = 0,$$

$$x = 0, \quad 0 \leq y < y_B \text{ and } y > y_B, t > 0 \quad (4b)$$

$$\frac{\partial H(x, y, t)}{\partial x} = 0, \quad x = \frac{L}{2}, t > 0$$

### Numerical Solution

Richard's equation is solved numerically using fully implicit finite volume method



**Figure 2.** Schematic representation of 5 point grid system.

with five points along with Picard iteration scheme (Figure 2).

By integrating Equation (1), on the finite volume, the following equation is obtained:

$$H_{i,j}^{n+1,m+1} = \frac{a_i H_{i+1,j}^{n+1,m+1} + b_i H_{i-1,j}^{n+1,m+1} + c_i H_{i,j+1}^{n+1,m+1} + d_i H_{i,j-1}^{n+1,m+1} + e_i}{g_i} \quad (5)$$

where:

$$a_i = \frac{k_{xe}^{n+1,m} \Delta y \Delta t}{\Delta x}, \quad b_i = \frac{k_{xw}^{n+1,m} \Delta y \Delta t}{\Delta x}$$

$$b_i = \frac{k_{xw}^{n+1,m} \Delta y \Delta t}{\Delta x}, \quad d_i = \frac{k_{yn}^{n+1,m} \Delta x \Delta t}{\Delta y} \quad (6)$$

$$e_i = C_{i,j}^{n+1,m} H_{i,j}^{n+1,m} \Delta x \Delta y - \left[ \theta_{i,j}^{n+1,m} - \theta_{i,j}^n \right] \Delta x \Delta y$$

$$f_i = C_{i,j}^{n+1,m} \Delta x \Delta y$$

$$g_i = a_i + b_i + c_i + d_i + f_i$$

In which  $m$  is the iteration number;  $n$  the time step;  $i$  and  $j$  are the node number for  $x$  and  $y$  directions respectively;  $C_{i,j}^{n+1,m}$  is the moisture characteristic curve slope corresponding to the node  $(i,j)$  at  $(n+1)_{th}$  time step and  $m_{th}$  iteration,  $H_{i,j}^{n+1,m}$  is the hydraulic head at the node  $(i,j)$  at  $(n+1)_{th}$  time step and  $m_{th}$  iteration, and  $k_{yn}^{n+1,m}$  and  $k_{xe}^{n+1,m}$  are the vertical and horizontal unsaturated hydraulic conductivities respectively in points  $N$  and  $E$  at  $(n+1)_{th}$  time step and  $m_{th}$  iteration. Other notations are defined in a similar way.

Equation (4) is solved by Picard iteration method using the following algorithm:

1. Potential values are assumed for all nodes except for boundaries.

2.  $a_i$  to  $g_i$  coefficients are calculated.
3. Boundary conditions are applied.
4. Equation (4) is applied for all nodes with unknown potential as well as for its 4 neighborhood nodes, this procedure being applied for rows beginning from down left (Figure 2).

Through iterating stages (2) to (4) until convergence is achieved, all potentials would be determined.

### Model Verification

The following steps are carried out for model verification:

- 1) When water table at drain location is assumed to be equal to initial water table, the water table predicted by the model is even all over the water table profile.
- 2) The proposed model results confirm the Fipps *et al.* (1986) finite element model's ones.

### Optimization Procedure

Optimization procedure is necessary for estimating the different parameters involved, soil moisture characteristic curve and unsaturated hydraulic conductivity function parameters, within the inverse technique context. Most of the nonlinear optimization methods could have been used in this study. Box (1966) tested most of the optimization methods for functions with 2, 3, 5, 10, and 20 independent variables and stated that Powell's method performance is more appropriate than that of the other methods. Thus, this method was selected in the current study. The objective function identified in the computer code in this research is:

$$F = \sum_{i=1}^n (h_i^m - h_i^c)^2 \quad (7)$$

where  $h_i^m$  and  $h_i^c$  are the measured and calculated water table depths (hydraulic heads) above drains for point  $i$  respectively,

while  $n$  being the number of the measured points.

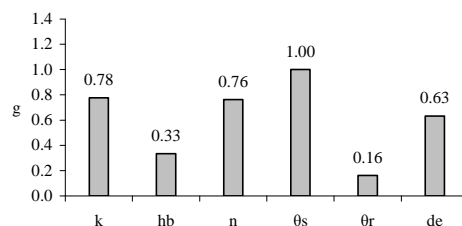
### Model Calibration

In order to calibrate the model, experimental data are needed and therefore a laboratory model was set up. The laboratory model involved a drainage model made up of metal and Plexiglas equipped with piezometers for piezometric potential measurements. The physical model was filled with soil and left submerged for more than a month so that a stable soil structure could be established. In order to assess the necessary parameters, the drain outlet was closed and submergence conducted. Then the outlet was opened and the drawdown of water table profile with respect to time measured. Water height in piezometers (hydraulic head) was measured with time. Vertical and horizontal saturated soil hydraulic conductivities were determined using the falling head method, the obtained results being 9.65, and 8.86  $\text{cm day}^{-1}$  respectively. For a measurement of soil moisture curve, pressure and ceramic plate membranes were employed. The data obtained by Pendy *et al.* were employed for the calibration process. In their research work, water table profiles and soil hydraulic conductivity of the laboratory model have been measured (Pendy *et al.*, 1992).

In this research, part of the water table data was used for calibration while the rest for verifying the obtained results. Calibration process was carried out by running the optimization computer program for water table heights, and hydraulic heads for various locations and related times, finally the parameter values being determined.

### Sensitivity Analysis

By conducting sensitivity analysis, the effect of model input parameters on output, (water table profile) is determined. This process gives a good insight into the impact



**Figure 3.** Values of  $\gamma$  for model input parameters.

of each parameter on the results and the applicability of the optimization process. Hill's method (Hill, 1998) was employed for calculating the sensitivity coefficient ( $\gamma$ ) for each of the input parameters. Parameters with  $\gamma$  less than 0.1 cannot be estimated through the inverse technique. Figure 3 shows the sensitivity coefficient for different input parameters where  $\theta_s$  and  $\theta_r$  are saturated and residual soil moisture contents, respectively. As shown in this Figure, sensitivity coefficient for all parameters is greater than 0.1 and therefore it is concluded that the parameters can be calculated using the inverse technique.

### Calibration as with Laboratory Data

As indicated previously, the purpose of calibration is to estimate optimal values for the involved parameters using laboratory data within the inverse technique program. The results are shown in Tables 1 and 2.

### Calibration with Pendy *et al.* Data

Tables 3 and 4 show calibration results when Pendy *et al.*'s data is employed.

### Verification

Two sets of data, laboratory and Pendy *et al.*'s, have been employed in the verification process. Using the soil hydrodynamic parameters obtained through the calibration

**Table 1.** Estimated and measured values of soil characteristic curve parameters.

Parameter	Value	
	Estimated	Measured
$h_b$ (m)	0.0513	0.071
$\lambda$	0.0101	0.023
$\theta_s$	0.398	0.418
$\theta_r$	0.097	0.0521

**Table 2.** Estimated and measured values of horizontal and vertical unsaturated hydraulic conductivity parameters.

Parameter	Value	
	Estimated	Measured
$K_{sx}$ ( $\frac{m}{day}$ )	0.101	0.0965
$K_{sy}$ ( $\frac{m}{day}$ )	0.0886	0.0804
$h_b$ (m)	0.0513	0.071
$\eta$	2.03	2.069

procedure water table profiles, not employed in the calibration procedure, were predicted for the two verification cases.

### Verification Using Laboratory Data

Using the soil hydrodynamic parameters obtained through the calibration procedure, water table profiles were predicted for 90 and 150 minutes after drainage started. The water table profile, hydraulic heads, after a lapse of 90 minutes is shown in Figure 4. Also water table variations with time, calculated through measured and estimated parameters, are shown in Figure 5.

### Verification Using Pendy *et al.* Data

For the verification, calculated and measured water table profiles after a 3 day lapse from drainage beginning, using Pendy *et al.* data, are shown in Figure 6. Also water table variations midway between drains with

**Table 3.** Values of constant parameters of soil moisture characteristic curve, using Pendy *et al.* (1992) data.

Parameter	Value	
	Estimated	Measured
$h_b$ (m)	0.308	-
$\lambda$	0.044	-
$\theta_s$	0.4	0.44
$\theta_r$	0.051	-

**Table 4.** Values of constant parameters of horizontal and vertical unsaturated hydraulic conductivity function, using Pendy *et al.* (1992) data.

Parameter	Value	
	Estimated	Measured
$K_{sx}$ ( $\frac{m}{day}$ )	0.035	0.0375
$K_{sy}$ ( $\frac{m}{day}$ )	0.04	0.04
$h_b$ (m)	0.308	-
$\eta$	2.132	-

respect to time are shown in Figure 7. As the measured soil moisture curve values were not available in Pendy *et al.*'s research, therefore prediction of water table profiles using measured parameters did not become possible.

To investigate and quantify the unsaturated vertical flow to drains using the estimated parameters, pressure potential magnitudes at different distances from drains (0.5, 5 and 10 days following the beginning of drainage) beginning are shown in Figure 8. In this Figure, zero pressure potential indicates water table. Figure 9 shows vertical and horizontal water velocity components as predicted by the model.

To evaluate the model statistically, such indices as RMSE (Root Mean Square Error), ME (Modeling Efficiency), EF (Efficiency) and MAPE (Mean Absolute Percentage) were employed. These indices can be expressed as follows (Homaei *et al.*, 2002):

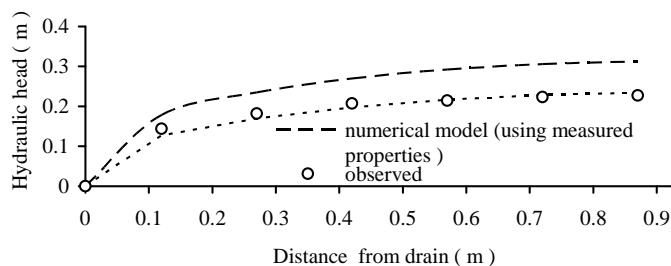


Figure 4. Observed and computed profile of water table, 90 minutes after initiation of drainage.

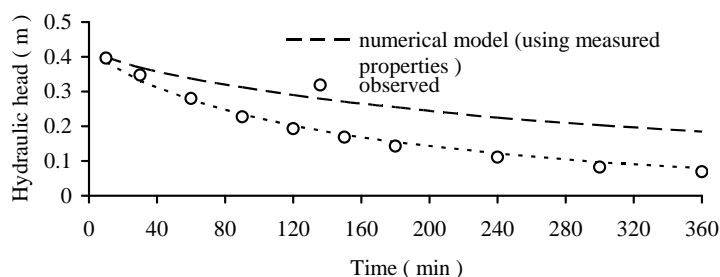


Figure 5. Observed and computed water table midway between drains.

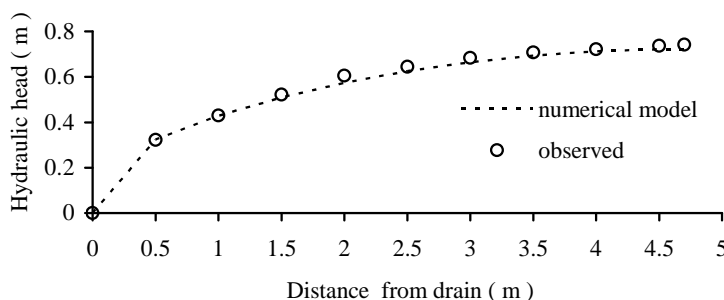


Figure 6. Observed and computed based profile of water table, 3 days after initiation of drainage.

$$RMSE = \left[ \frac{\sum_{i=1}^n (h_i^c - h_i^m)^2}{n} \right]^{1/2} \tag{8}$$

$$ME = \max |h_i^c - h_i^m|_{i=1}^n \tag{9}$$

$$EF = \frac{\sum_{i=1}^n (h_i^m - \bar{h})^2 - \sum_{i=1}^n (h_i^c - \bar{h})^2}{\sum_{i=1}^n (h_i^m - \bar{h})^2} \tag{10}$$

$$MAPE = \frac{\sum_{i=1}^n \left| \frac{h_i^c - h_i^m}{h_i^m} \right|}{n} \tag{11}$$

where  $h_i^c$  and  $h_i^m$  are the calculated and measured hydraulic heads, respectively,  $\bar{h}$  is the average hydraulic head and  $n$  the number of measured points. The indices related to the model are shown in Tables 5 and 6.



### RESULTS AND DISCUSSION

Figures 4 to 7 and Tables 5 to 6 imply good agreement between measured and predicted water table hydraulic heads. The profiles predicted by employing the

estimated unsaturated soil hydrodynamic characteristics are much more indicative than those predicted while using the measured ones, indicating the importance of measurement error effects on the results.

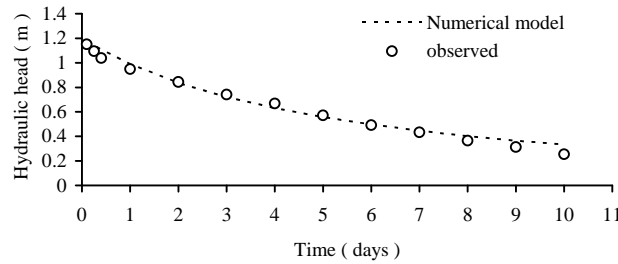


Figure 7. Observed and computed based water table, midway between drains.

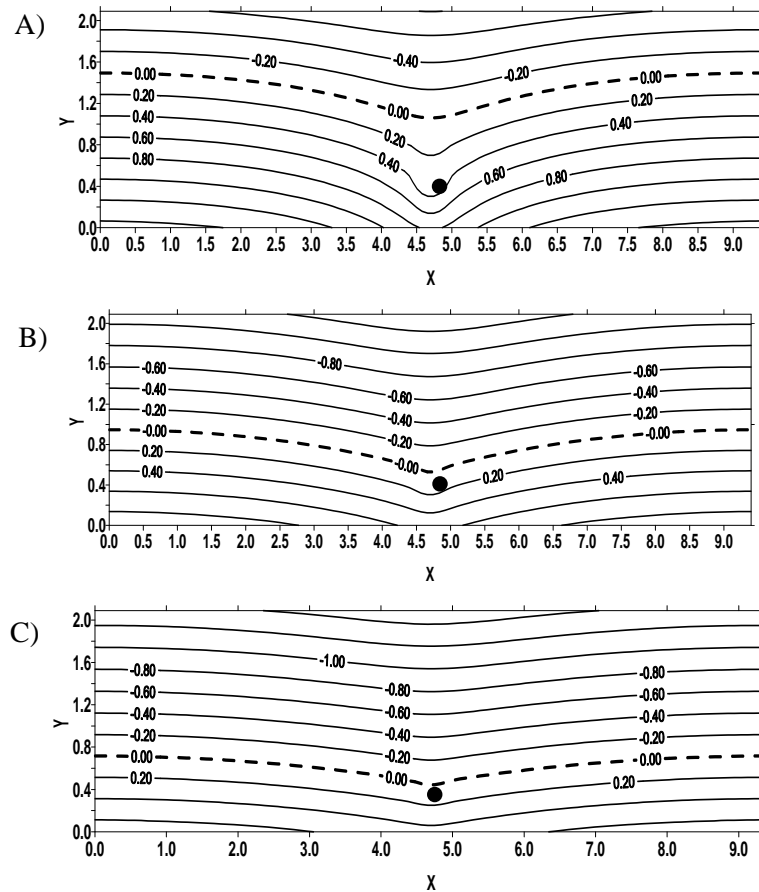
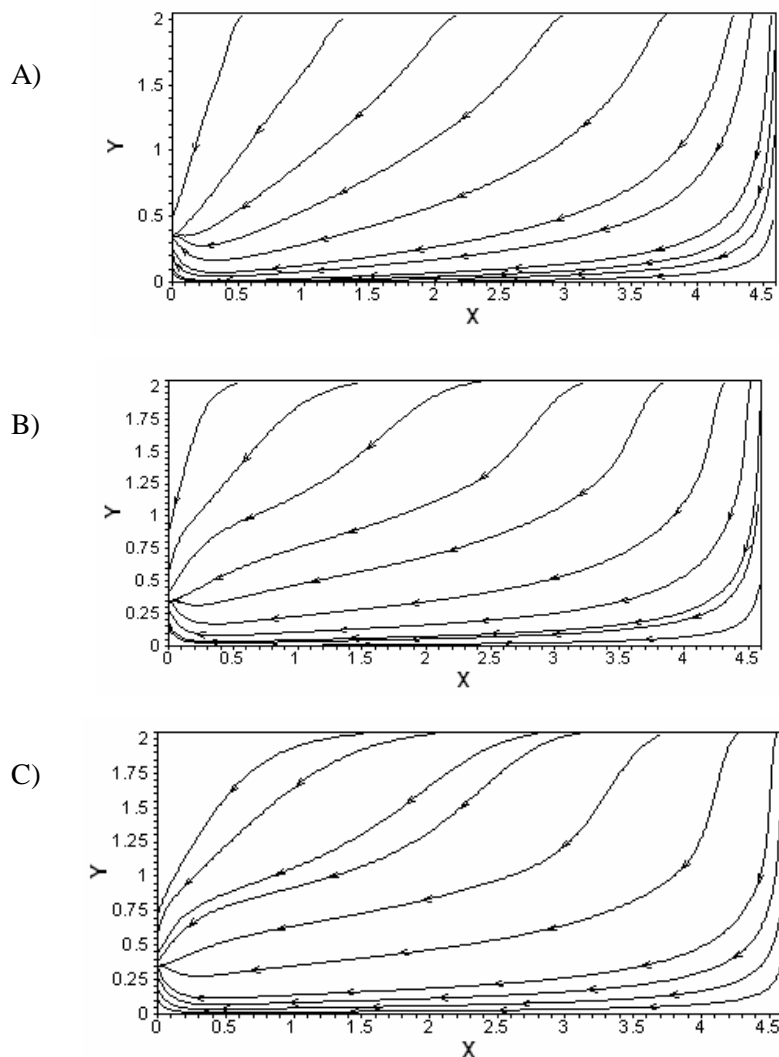


Figure 8. Isobar of soil at: (A) 0.5, (B) 5 and (C) 10 days after initiation of drainage.





**Figure 9.** Pathlines towards drains at: (A) 0.5, (B) 5 and (C) 10 days after initiation of drainage.

The present research indicates the advantage of the proposed method in estimating the unsaturated soil hydrodynamic characteristics using water table profile variations, it being concluded that flow condition is playing an important role in an estimation of the parameters.

Figures 8 and 9 indicate that unsaturated zone plays an important role in flow close to drains and its effect becomes more significant with a lapse of time and as the unsaturated zone becomes extended. Therefore, the proposed 2-D model of

saturated-unsaturated drainage model can serve as a proper tool to fulfill the objectives of this research.

Finally, the results indicate that linking the numerical model solution to the inverse problem technique introduces an effective tool capable of estimating unsaturated soil hydrodynamic characteristics, necessary in both design and management of drainage systems.

**Table 5.** Values of statistical indices (Calculated using measured data).

Statistical indices	Numerical model ( Using estimated properties )	Numerical model ( Using measured properties )
RMSE ( m )	0.0099	0.0847
ME ( m )	0.0271	0.121
EF ( % )	97.65	73.1
MAPE ( % )	6.8	75.81

**Table 6.** Values of statistical indices (Calculated using Pendy *et al.* (1992) data) .

Indices	Value
RMSE ( m )	0.0275
ME ( m )	0.075
EF ( % )	97.71
MAPE ( % )	5.7

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## تخمین پارامترهای هیدرودینامیکی غیر اشباع خاک با استفاده از مسأله معکوس

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### چکیده

شبه‌سازی ریاضی جریان به طرف زهکش‌ها مهمترین قدم در طراحی و مدیریت سیستمهای زهکشی به شمار می‌آید. اگر چه مدل‌های زهکشی مختلفی توسط محققان بسط و توسعه داده شده است. اما اکثر این مدل‌ها تنها قادر به مدل‌سازی جریان اشباع به طرف زهکش‌ها می‌باشند، در حالی که در سیستمهای زهکشی جریان به طور توأم در دو ناحیه اشباع و غیر اشباع خاک اتفاق می‌افتد. در این تحقیق با منفصل سازی معادله دیفرانسیل حاکم بر جریان اشباع- غیر اشباع به طرف زهکش و اعمال شرایط مرزی مناسب، مدلی عددی جهت پیش‌بینی نوسانات سطح ایستابی در اطراف زهکش بسط داده شد. با انتخاب الگوریتم بهینه‌سازی مناسب و اتصال آن به مدل عددی، مدل معکوس مناسب جهت برآورد مقادیر میانگین توابع آبی غیر اشباع خاک طراحی گردید. با طراحی و ساخت مدل فیزیکی زهکشی در آزمایشگاه، داده‌های مورد نیاز جهت واسنجی و آزمون مدل پیشنهادی اندازه‌گیری شد. از داده‌های آزمایشگاهی محققان دیگر نیز برای ارزیابی دقت پیش‌بینی مدل پیشنهادی استفاده شد. نتایج حاصل از تحقیق نشان داد، با استفاده از مدل معکوس پیشنهادی مقادیر متوسط ویژگیهای هیدرودینامیک غیر اشباع خاک را با دقت بالایی می‌توان برآورد کرد. با استفاده از مقادیر برآوردی از مدل می‌توان نوسانات سطح ایستابی در اطراف زهکش را با دقت بیشتری پیش‌بینی کرد.