
Sh. Rafiee1*, M. Kashaninejad2, A. R. Keyhani1, and A. Jafari1

ABSTRACT

Drying is an important operational step in pistachio processing. There is a decrease in moisture content of pistachio nuts from 56-57% (d.b.) to 5-6% during the drying process. Drying conditions affect the quality of dried pistachio nuts, therefore, this calls for an accurate description of the drying trend in the process. In this study finite element formulation and solution of diffusive moisture transfer equation was presented to improve the drying simulation of nut as an axisymmetric body. The Fick's diffusive model was solved with effective moisture diffusivity of $5.24 \times 10^{-10} \text{ m}^2 \text{s}^{-1}$ for 55°C and $7.01 \times 10^{-10} \text{ m}^2 \text{s}^{-1}$ for 70°C. For experimentation, thin layers of pistachio nut, ‘Ohadi’ variety, were dried at high drying air temperatures (55 and 70°C), three replications for each treatment, along with drying air velocity and relative humidity of 0.5 m s$^{-1}$ and 20%, respectively. Good agreement was observed when the output of model was compared with the experimental data. Mean Relative Deviation (MRD) calculated for the model and the experimental data for the air temperatures 55 and 70°C, were found to be 6.2% and 8.1%, respectively.

Keywords: Drying simulation, Finite element method, Moisture diffusivity, Pistachio nut.

INTRODUCTION

Pistachio nut is an edible seed of the pistachio tree. Several species of the genus Pistacia are referred to as pistachio, but only the fruits of Pistacia vera attain sufficiently large size to be acceptable to consumers as edible nuts (Shokraii and Esen, 1988). The nuts are consumed as confectionary ingredient, or snack. Because of the deep green color of pistachio kernels, it is highly favored in ice cream and in pastry industries. Pistachio is cultivated in the Middle East, United States and Mediterranean countries. Iran is one of the biggest producers and exporters of pistachio nuts. More than 200,000 tones of pistachio nuts are annually produced in Iran (Anon., 2001).

A high quality of pistachio nuts largely depends on its fast and efficient handling and drying operations. Kader (1982) pointed out that pistachio nuts benefit from a highest quality when harvested fully mature and dried immediately after harvest. During the drying process, nuts can undergo fully mature reactions, which cause a loss of nutritional value and unfavorable enzymatic activities. In comparison with other food products, studies on drying of pistachio nuts are very limited. It would therefore be helpful and necessary to study the simulation of the drying process in pistachio nuts. This, however, requires a knowledge of such nut kernel drying characteristics as under different drying conditions. Numerical methods that describe drying mechanisms of foods can provide a clue to the required temperature and moisture requirements (Rafiee and Kashaninejad, 1

---

1 Faculty of Biosystem Engineering, University of Tehran, Islamic Republic of Iran.
2 Gorgan University of Agricultural Sciences and Natural Resources, Gorgan, Islamic Republic of Iran.
* Corresponding author, e-mail: shahinrafiee@ut.ac.ir
2005; Rafiee et al., 2005; Haghhighi and Segerlind, 1988). Among numerical methods, finite element has found wide application due to its high effectiveness (Nieber, 1983).

Much work has been done to simulate the temperature, moisture content as well as stress distributions inside single grain kernels (Gustafson et al., 1979; Haghhighi and Segerlind, 1988; Lague and Jenkins, 1991; Irudayaraj and Haghighi, 1993; Jia et al., 2000a, b). The mathematical models applied by these authors have been similar but the geometrical models somewhat different. It is an agreed principle that temperature distribution within a geometrical model (spherical or cylindrical coordinate systems) is not uniform. Simultaneous equations for moisture and heat diffusion are necessary to describe moisture movement within agricultural products. The drying models presented by most researchers are very similar to the modified Luikov’s equations used by Husain et al. (1973).

Miketinac et al. (1992) used the finite element method to solve the non-linear coupled systems of two partial-differential equations describing the thin layer drying process of grain and calculated the heat and mass transfer coefficients using the inverse method. Casada and Young (1994) developed a model to predict heat and moisture transfer for long-term moisture migration in peanuts due to natural convection and diffusion in an arbitrarily shaped porous media. Jia et al. (1996, 2000) performed simulation of temperature and moisture fields inside a maize kernel through an application of finite element method. Yang et al. (2002, 2003) applied finite element method to predict intra–kernel moisture content distribution during drying and tempering processes in rice and examined the relations between moisture content gradients and head rice yield trends during drying and tempering processes. Wu et al. (2004) developed a mathematical model describing the simultaneous heat and mass transfer in a single kernel of rice in the drying process.

The objective of this research was to develop the finite element formulation and to present the solution to a set of coupled conductive heat and diffusive moisture transfer equations for a single pistachio nut. The model considers the pistachio nut as an axisymmetric body. The model was verified by the experimental data obtained during the thin layer drying process.

MATERIALS AND METHODS

Theoretical Formulation and Finite Element Analysis

For nut drying simulation, Fick’s diffusive equation describing the mass transfer process has been extensively applied (Jia and Sun, 2000; Gastón et al., 2002; Yang, 2002):

$$\frac{\partial M}{\partial t} = \text{div}(DM)$$

where $M$ is the moisture content d.b. (kg kg$^{-1}$); $D$ is the diffusion coefficient (m$^2$ s$^{-1}$); and $t$ is time (s). In moisture diffusion during the drying process, the surface of the nut exchanges heat with the environment by convection while the internal part is heated by conduction. If assuming that the moisture diffuses to the outer boundary of the kernel in liquid form and that the evaporation takes place at the surface of the nut, besides Equation (1), the heat transfer equation for pistachio nut should also be given as:

$$\rho c \frac{\partial T}{\partial t} = \text{div}(k\nabla T) + \frac{L}{\rho} \frac{\partial M}{\partial t}$$

where $\rho$ is the density (kg m$^{-3}$); $c$ is the specific heat (J kg$^{-1}$ K$^{-1}$); $T$ is the temperature (K); $k$ is the thermal conductivity (W m$^{-1}$ K$^{-1}$); and $L$ is the latent heat of vaporization of water (J kg$^{-1}$), all properties being given for a single nut. The initial conditions for the governing equations at $t=0$ are:

$$M(x, y) = M_0$$

$$T(x, y) = T_0$$

where $x$, $y$ are directions and for $t>0$ and the boundary conditions at the surface of the kernel are:
\begin{align*}
D \left( \frac{\partial M}{\partial x} l_x + \frac{\partial M}{\partial y} l_y \right) + h_m(M - M_\infty) &= 0 \\
\left( \frac{\partial T}{\partial x} l_x + \frac{\partial T}{\partial y} l_y \right) + h(T - T_\infty) &= 0
\end{align*}

(5)

(6)

where \( l_x, l_y \) are the direction cosines of the outward drawn normal to the boundary; \( h_m \) is the surface mass transfer coefficient (m s\(^{-1}\)); \( M_\infty \) is the moisture content of the ambient air d.b. (kg kg\(^{-1}\)); \( h \) is the convection heat transfer coefficient (W m\(^{-2}\) K); and \( T_\infty \) is the ambient temperature (K).

To solve the problem, coupled heat and mass transfer governing Equations (1 and 2) were taken into consideration but due to the importance of the simulation of the moisture change during the drying process of the nut, the result of the mass transfer governing equation (Equation 1) is shown only. The assumptions for the drying model are: isotropic and homogeneous material, no shrinkage, liquid diffusion only and axisymmetric shape.

The two-dimensional mathematical model is chosen so that the new coordinates can form egg-shaped surfaces. Equation (1) was rewritten as:

\[
\frac{\partial M}{\partial t} = D \left( \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \right)
\]

(7)

Using the variation method, the functions of the partial differential equations from Equation (1) can be written as:

\[
J = \int \left[ \frac{1}{2} \frac{\partial}{\partial t} \left( \frac{\partial M}{\partial x} l_x^2 + \frac{\partial M}{\partial y} l_y^2 \right) + 2 \frac{\partial M}{\partial t} \right] dV + \int \frac{h_m}{2}(M - M_\infty)^2 dS
\]

(8)

A nut kernel can be divided into \( m \) triangular elements, then the moisture within each element, \( M^{(e)} \), can be approximated by the linear interpolation polynomials, that is:

\[
M^{(e)} = N_i X_i + N_j X_j + N_k X_k
\]

where \( N \) is the shape function, The minimum values of Equation (8) are solved:

\[
\frac{\partial M}{\partial t} - \int_S m \left[ \frac{d}{dx} \frac{\partial M}{\partial x} l_x + \frac{d}{dy} \frac{\partial M}{\partial y} l_y + \frac{\partial M}{\partial t} \right] = 0
\]

(9)

Equation (9) can be simplified and expressed in a matrix form:

\[
K \{X\} + C \{ \dot{M} \} - F = 0
\]

(10)

where \( K \) is the element mass conductance matrix, \( C \) is the element mass capacitance matrix and \( F \) the element mass force vector. Coefficients of the above equation were integrated and rewritten (Segerlind, 1984):

\[
K = 2\pi dR_1 \left[ \begin{array}{ccc}
\frac{b_1}{c_1} + c_1 & b_1 & c_1 \\
b_1 & b_1 + c_1 & b_1 + c_1 \\
b_1 & b_1 + c_1 & b_1 + c_1
\end{array} \right] + \frac{2\pi dR_1}{6} \left[ \begin{array}{ccc}
(r_1 + r_2) & (r_1 + r_2) & 0 \\
(r_1 + r_2) & (r_1 + r_2) & 0 \\
0 & 0 & 0
\end{array} \right]
\]

(11)

\[
C = \frac{2A}{30} \left[ \begin{array}{ccc}
6r_1 + 2r_2 + 2r_3 & 2r_1 + 2r_2 + r_3 & 2r_1 + r_2 + 2r_3 \\
2r_1 + 2r_2 + r_3 & 2r_1 + 6r_2 + 2r_3 & r_1 + 2r_2 + r_3 \\
2r_1 + r_2 + 2r_3 & r_1 + 2r_2 + r_3 & 2r_1 + 2r_2 + 6r_3
\end{array} \right] + \frac{2\pi dR_1 M_{ij}}{6} \left[ \begin{array}{ccc}
2 & 1 & 0 \\
1 & 2 & 0 \\
0 & 0 & 1
\end{array} \right]
\]

(12)

\[
F = \frac{2\pi dR_1 M_{ij}}{6} \left[ \begin{array}{ccc}
2 & 1 & 0 \\
1 & 2 & 0 \\
0 & 0 & 1
\end{array} \right]
\]

Using the forward finite difference approximation, \( \frac{\partial M}{\partial t} \) may be written as:

\[
\frac{\partial M}{\partial t} = \frac{M^{n+1} - M^n}{\Delta t}
\]

(14)

and Equation (10) rewritten as:

\[
\left( K + \frac{C}{\Delta t} \right) M^{n+1} = C M^n + F
\]

(15)

A computer program for a two-dimensional transient field problem such as the one described by Equation (15) was written by Segerlind (1984). The effect of moisture content for each time step was modified for use in axially symmetric triangular elements. This program first solves Equation (15) for given initial nodal values. For every time step \( \Delta t \) and a given set of nodal values \( \{M\}_1 \), a set of nodal moisture values \( \{M\}_{i+1} \) are obtained and stored. A code was written in FORTRAN 90 to solve Equation (15).

Discretization of a single nut is shown in Figure 1. Due to the symmetrical shape, only one quarter of a nut kernel is shown. For
clarity, the figure shows only 25 elements but in reality the number of elements totalled 1296 where no more significant change in the accuracy of the model was observed.

**Thin Layer Drying Experiments**

**Sample Preparation**

‘Ohadi’ is the major pistachio nut variety grown in Iran. Therefore, this cultivar was picked up for the study. Nuts were received when at an average moisture content of 4-5% (d.b.). They were stored in a refrigerator in sealed double layers of polyethylene bags. Before conducting the experiment, moisture content of the nuts was raised to 56-58% (d.b.) by intermittently adding a predetermined amount of distilled water in a sealed glass jar. After sealing the jar, and tumbling until all moisture was absorbed by the product, the samples were left for 10 days at 5°C to ensure equilibration. During this period they were mixed thoroughly, and regular intervals (Palipane and Driscoll, 1994; Pathak et al., 1991).

**Drying equipment and experimental procedure**

Figure 2 shows and schematic diagram of the air recirculating dryer unit. It consists of an air conditioning unit, vane axial circulating fan, drying chamber with scale-mounted trays, and a connecting duct system. The air conditioning unit (Bryant Manufacturing Model AH-213, BMA, Inc., Ayer, MA) consists of a conditioning chamber with dimensions of 711x711x965 mm, and with the exterior dimensions of 965x1425x965 mm. The air temperature (range of -17 to 200°C) and the relative humidity (range of 5–98% limited by 2°C dew point) are controlled by a Watlow microprocessor controller with a temperature accuracy of ±0.25°C and relative humidity accuracy of ±2%. The unit is equipped with a solid-state humidity sensor, a steam injection humidifier, and a twin-tower desiccant-type dehumidifier that can achieve low humidities down to 2%. Air is conditioned in the chamber at the set points of air temperature and relative humidity. The conditioned air is recirculated through the dryer unit by an axial fan (Model VA7D32, American Cool air Corporation, Jacksonville, FL) with a nominal rotational speed of 3200 rpm and a three-phase 220 V motor rated 1/12 horsepower. The fan speed is controlled by a variable electronic transistor inverter (Model VFS7, Toshiba Corporation, Japan) with a frequency range of 0.5–80 Hz. (cf. 60 Hz for ‘normal speed operation’). The experiments were conducted at two air temperatures (55 and 70°C), one air velocity (0.5 m s⁻¹) and one relative humidity (20%) and three replications for each treatment. To minimize experimental error, each drying test was performed in triplicate. Before the start of each drying run, a 250 g sample was removed from refrigerator and placed in a plastic bag in the laboratory to bring the temperature of pistachio nuts to the room temperature. For each run the equipment was allowed for at least two hours to stabilize at the specified air conditions before the test began. Then, the pistachio nuts were spread in a thin layer on drying trays and placed in the drying chamber. The ambient, upstream and downstream dry bulb temperatures, air...
relative humidity, air velocity and sample weights were continuously monitored and recorded every once 60 seconds. Drying was continued until the moisture content of the sample reached 5% (d.b.). After each drying experiment, the sample was oven-dried at 103±2°C, the average moisture content of the samples being calculated as based on the initial mass and final moisture content of the samples (Kashaninejad et al., 2003).

The derived governing equations were applied to simulate the drying process of a pistachio nut. Results were compared to those given in the literature. Equations 1 and 2 were solved simultaneously but here only the mass transfer results are presented. Experimental and theoretical results were compared to find the accuracy of the model.

The simulation of pistachio nut drying was compared with experimental results by the Mean Relative Deviation (MRD) as follows (Abalone et al., 2000):

$$MRD = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \frac{M_{\text{pre},i} - M_{\text{exp},i}}{M_{\text{exp},i}} \right)^2 \right]^{1/2}$$

(16)

where $M_{\text{exp},i}$ is the nth experimentally observed moisture ratio, $M_{\text{pre},i}$ the nth predicted moisture ratio, and N the number of observations (Sarsavadia et al., 1999).

RESULTS AND DISCUSSION

Verification of the Finite Element Model

In this study, the initial moisture content of pistachio nuts in air temperatures of 55 and 70°C were 56.3% (d.b., kg kg⁻¹) and 56.9% (d.b., kg kg⁻¹), respectively. During the drying process, moisture was determined by every one minute intervals.

Moisture content of the product is a function of moisture diffusion in the kernel only. Hence, h and $h_m$ will not contribute to the model as important coefficients. No values were found in the literature for h and $h_m$ for pistachio nuts, therefore, corresponding values for soybean kernels were used ($h = 60.0 \text{ W m}^{-2} \text{ °C}$ and $h_m = 0.05 \text{ m} \text{s}^{-1}$) were taken into consideration while 10 folds of either more or less than the values were also tested with very minor differences observed. This confirms the fact that these coefficients played minor roles in the drying process (Haghighi and Segerlind, 1988).

A computer code for predicting the moisture fields inside the pistachio nut quarter was developed employing Fortran-90 language.
The Fick’s diffusive model was solved with effective moisture diffusivity of $5.24 \times 10^{-10}$ m$^2$ s$^{-1}$ for 55°C and $7.01 \times 10^{-10}$ m$^2$ s$^{-1}$ for 70°C (Kashaninejad et al., 2007). A comparison between the simulated and the measured average moisture contents under two different thin-layer drying temperatures is shown in Figures 3 and 4. It can clearly be seen that the simulated results well agree with the measured values.

Figure 3 shows the simulated and measured variation of the average moisture with drying time under air temperature of 55°C. The simulated moisture contents between 25 and 200 minutes were a little lower than those of the measured values. The simulated moisture curve and experimental variation of moisture with air temperature at 70°C are shown in Figure 4. This figure shows that the simulated values between 10 and 140 minutes and between 140 and 200 minutes were either slightly lower or slightly higher than the measured values, respectively. It seems from Figure 4, that if the time in the drying process is extended, the same trend as in Figure 3 would be expected. Similar results have been reported for wheat (Gastón et al., 2002; Jia and Sun, 2000), peanut (Casada and Young, 1994), maize (Jia et al., 1996, 2000) and rough rice (Yang et al., 2002, 2003).

The MRD between simulation values for moisture diffusivities $5.24 \times 10^{-10}$ m$^2$ s$^{-1}$ for 55°C and $7.01 \times 10^{-10}$ m$^2$ s$^{-1}$ for 70°C (Kashaninejad et al., 2007) and experimental thin layer drying at 55 and 70°C were 6.2%
and 8.1%, respectively.

**Moisture Distributions**

Haghighi and Segerlind (1988) and Fortes *et al.* (1981) have also reported that a combination of moisture and temperature gradients would produce greater stress levels in the nut. In order to examine the moisture and thermal stresses, it is important to know the temperature and moisture distributions in the kernel, particularly at the early stages of the drying process.

Figures 5 and 6 show the moisture content distribution inside the kernel at six selected times: 5, 50, 100, 200, 300, and 400 minutes under the air temperatures of 55 and 70°C. The moisture content of the outermost layer of the kernel fell to the equilibrium level within a short time, while moisture in the central part of the kernel was still at a relatively high level even after 35 and 100 minutes of drying and at

![Figure 5](image-url)

**Figure 5.** Moisture distribution at selected drying times for a pistachio nut (1/4 kernel shown) at initial moisture content of 56.3 (d.b.%) and T= 55 °C.

![Figure 6](image-url)

**Figure 6.** Moisture distribution at selected drying times for a pistachio nut (1/4 kernel shown) at initial moisture content of 56.9 (d.b.%) and T= 70 °C.
air temperatures of 70 and 55°C, respectively. Moisture content differences between the center part and the surface were high at the beginning of the drying process, but dropped gradually over time. Similar trends were reported for the moisture distribution for different drying times within a single wheat kernel by Jia and Sun (2000), maize kernel by Neményi et al. (2000) and barley by Haghighi et al. (1990).

CONCLUSION

A Finite Element Model (FEM) for pistachio nut drying has been developed and verified through a thin layer drying process. The predicted average moisture values agreed well with the experimental results, verified by low MRDs. The results suggest that using the FEM can properly predict the moisture content distribution inside the pistachio nut from which the moisture gradient as well as stress can be calculated.

NOTATION

\begin{align*}
\rho & \quad \text{Density Kg m}^{-3} \\
\text{exp} & \quad \text{Experimental data} \\
\text{pre} & \quad \text{Predicted data} \\
\infty & \quad \text{Ambient}
\end{align*}

REFERENCES

شبیه سازی انتقال جرم در مدت زمان خشکش دند به روش اجزای محدود در پست رقم اوغدی

ش. رفیعی، م. کاشانی زاده، ع. ر. گهاتی، و غ. جعفری

چکیده

خشک کردن یکی از فرآیندهای مهم در فراوری محصول پسته می باشد. در طی فرایند خشک کردن پسته، رطوبت از 7 تا 40 درصد (شرایط خشک) به 5 تا 4 درصد تقلیل می یابد. اگر بازاب یا در سرعت و کارایی درجه‌بندی و شرایط خشک کردن آن استقامت دارند، شرایط خشک کردن سپرهای و هوا خشک کردن، سرعت هوا، رطوبت نسبی و محقای رطوبت اولیه بروز کیفیت شیمیایی و میکروبیولوژی پسته خشک شده تأثیر دارد. بنابراین فراوانی خشک کردن باید مطالعه شده و تحت کنترل درآید. در این منظور لازم است تا مکانیزم خشک شدن با دقت تشریح گردد. در تحقیق حاضر فرمول سازی المان محدود و حل معادلات انتقال سرعت رطوبت جهت بهبود خشک کردن پسته ارائه شده است. مدل به صورت پخش رطوبت اصلاح شده (1×10⁻² m² s⁻¹) برای دمای 5 درجه سانتی‌گراد و (70%×10⁻⁴ m² s⁻¹) برای دمای 70 درجه سانتی‌گراد حل شد. در آزمایشات جریان هوا داغ (55 و 60 درجه سانتی‌گراد) بر روی یک لایه نازک از پسته رقم اولیه عبور داده شد. قطعیت رطوبت سرعت هوا خشک شدن و رطوبت نسبی بر پرسته 1/5 و 20/20 درصد، میانگین از طول خشک کردن اندازه‌گیری می گردید. انتقال خوری بین داده‌های آزمایشگاهی در مقایسه با مدل نوازه ی بی‌کلاس آزمایشگاهی، برابر 1/6 درصد و 8/0 درصد بودند. این تحقیق نشان می دهد که مدل‌های آزمایشگاهی با مدل نوازه‌ای یکین کنندگی است. از مدل برای شبیه سازی رطوبت در خشک کردن پسته استفاده شد. از نظر شبیه سازی رطوبت و گرادیان در داخل دانه مستقیم برای تحلیل نشان ترک کردن پسته استفاده می شود. همچنین از نتایج تحلیل اجزای محدود می توان برای ارزیابی کیفیت پسته و همچنین مطالعات شبیه سازی خشک کردن آن استفاده نمود.