Mathematical Modeling of Heat Transfer and Sterilizing Value Evaluation during Caviar Pasteurization

M. Khakbaz Heshmati*, M. Shahedi1, N. Hamdami1, M. A. Hejazi2, A. A. Motalebi3, and A. Nasirpour1

ABSTRACT

In this study, a numerical model was developed to investigate the two-dimensional heat transfer in a homogenous finite cylinder to predict the local temperature and sterilizing value during caviar pasteurization. A fixed grid finite difference method was used in the solution of heat transfer equations according to Crank-Nicolson’s scheme. The model was validated by comparison of the experimental temperature profiles during caviar pasteurization with the model predicted values (Correlation Coefficient> 0.99 and Root Mean Square Errors< 0.61°C). The cold spot location was at the radial center between the middle and top of the jar on the vertical axis. For caviar pasteurization, the required heating time for cold spot to reach the desired F-value ($F_{55^\circ°C} = 0.19$ min) was 128 minutes at 55°C and 37.63 minutes at 65°C. The results indicated that the developed model could be successfully applied to simulate the caviar thermal processing.

Keywords: Crank-Nicolson’s scheme, Thermal lethality.

INTRODUCTION

Caviar is an expensive delicacy with high nutritional factors (Altug and Bayrak, 2003; Bledsoe et al., 2003). The caviar from Russia and Iran, produced from roes of sturgeon fish, is known as the original caviar. There are over 20 species of sturgeon, while the Caspian Sea has six commercially valuable sturgeon species, four of which produce 90% of the world’s original caviar (Jelodar and Safari, 2006; Rezvani Gilkolaei, 2002). In the conventional processing, caviar contains 4–8% salt, which is added after sieving the eggs from the ovaries, with the better varieties generally containing less salt (Wang et al., 2008). Although the caviar production is carried out with the highest hygiene level, it is still possible to observe microbial contaminations. Altug and Bayrak (2003) analyzed microbiologically the caviar samples from Russia and Iran. They detected Coliforms, Escherichia coli, and yeast in the samples and reported that the standard plate count (SPC) and yeast counts were beyond the desired levels. Because of the high content of salt and the possibility of contamination in the conventional method of caviar processing, pasteurization in the convenient temperature and time is one of the best methods of having a good product quality. Caviar is heat labile and difficult to pasteurize successfully. It can only be heated to a temperature of 70°C without the eggs becoming dull and losing their color (Sternin and Dore, 1993). Therefore, irreversible
protein denaturation occurs between 70 and 80°C (Al-Holy et al., 2004). Industrial processes of pasteurization have to ensure the prolongation of food shelf life, while the quality of product would be preserved. The accomplishment of both purposes depends on process conditions with the assurance of the adequate temperature course during processing, where consideration of the temperature profiles within the product has great importance (Augusto et al., 2009; Plazl et al., 2006).

Heat transfer mechanisms in a canned food include conduction for solid foods, natural convection, especially for low viscosity liquid foods, convection plus conduction for liquid foods with solid particles, and convection followed by conduction for liquid foods containing starch or high viscosity modifiers (Chen and Ramaswamy, 2007). During thermal process, the temperature inside the food depends on time as well as on the position inside the food system. The design of the thermal process is, therefore, always based on the temperature course in that position in the can that receives the least intense heat treatment. Temperature profiles are determined using analytical and numerical solutions of partial differential equations governing the process (Kızıltas et al., 2010). Most of the heat transfer models can only be solved analytically for simple cases. Numerical methods are useful for estimating the thermal behavior of foods under complex but actual conditions such as variation in initial temperature, non-linear and non-isotropic thermal properties, irregular-shaped geometries, and time-dependent boundary conditions (Puri and Anantheswaran, 1993). Thus, for realistic and more complicated heat transfer problems, usually no analytic solution is available, and a numerical solution becomes mandatory. Numerical modeling is very useful in investigation, prediction, and optimization of food manufacturing, including thermal processing (Plazl et al., 2006). It can be used to produce much valuable information about the heating processes of foods under broad experimental conditions within a short time while traditional experiments can only be restricted to several special conditions due to experimental cost and time limit (Wang and Sun, 2003).

Several researches have been done on caviar with different subjects, such as determination of chemical composition (Duyar et al., 2008), proteins (Al-Holy and Rasco, 2006), fatty acid and volatile compounds (Caprino et al., 2008), organic and metal contaminants (Wang et al., 2008), and microbiological aspects (Altug and Bayrak, 2003; Jelodar and Safari, 2006). However, little research has been published on the thermal processing of caviar (Al-Holy et al., 2005). Al-Holy et al. (2004) studied thermal inactivation of *Listeria innocua* in salmon caviar using conventional glass and novel aluminum thermal-death-time tubes. Also, they determined dielectric properties of salmon and sturgeon caviar for microwave pasteurization (Al-Holy et al., 2005). Numerical modeling of thermal processing has been conducted with different food systems in the past (Akterian, 1995; Ghani et al., 1999; Kızıltas et al., 2010; Mohamed, 2007; Plazl et al., 2006). However, to our knowledge, no previous research has studied the thermal process modeling of caviar. Also, in the previous studies on heat transfer in the other canned foods with heat conduction assumption within the can, it can be seen that cold spot of can during heating have been determined at the geometric center. However, looking at the boundary conditions (the condition at the headspace is different compared to others, so there is no axial symmetry) indicates that cold point cannot be located at the geometric center. Headspace volume value of canned foods varies between 6-10% of the total volume. To our knowledge, no previous research has studied the impact of headspace content evolution on the heat transfer in canned food and the cold spot location.

This research is part of a study on pasteurization of caviar to provide an aid in the improvement of pasteurization design
and operation. The aims of current work are: (a) to develop a model for caviar pasteurization to allow prediction of temperature profile and sterilizing value, (b) to validate the theoretical model against experimental data, and (c) to determine the cold spot location as a function of headspace volume.

MATERIALS AND METHODS

Model Development

The caviar jar is modeled as a finite cylinder (Figure 1). The cross-section of the finite cylinder is divided into a fixed grid system in radial and vertical directions with different volume elements. The caviar container, at uniform temperature $T_o$, is placed into the water bath with a temperature $T_w$. The jar surface, directly exposed to water, is heated by convection. The heating of the surface creates a temperature gradient, which is the origin of forcing behind the heat transfer in the product toward the center of the container. Heat transfer inside the material can be treated as heat conduction, while heat transfer in the headspace of jar is carried out by natural convection or conduction. Resistance against heat transfer in the headspace is higher than in other boundaries.

To simplify the problem, the following assumptions were made in this work:

- Heat transfer is transient.
- Jar is assumed as a cylindrical geometry.
- Ax-symmetry, which reduces the problem from 3D to 2D.
- Container is impermeable against moisture, and mass transfer is negligible.
- Caviar is homogeneous and isotropic.
- Headspace is full of air.
- The initial temperature is uniform and constant.
- The convective heat transfer coefficient ($h$) and the surrounding water temperature are constant.
- Density changes due to temperature are negligible.

The heat transfer Equations (1-4), with the boundary conditions (5-8) form a complete mathematical model for the above-mentioned system.

Governing Equations

Heat Balance Equation

Two-dimensional transient heat conduction in a finite cylinder that has an isotropic and homogeneous material without heat generation is described by Fourier’s Equation (Incropera and DeWitt, 1990):

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r k \frac{\partial T}{\partial r} \right) \quad (1)$$

Figure 1. Schematic description of the system model.
The temperature dependence of specific heat and thermal conductivity are expressed by the following equations (Dalvi and Hamdami, 2011):

$$C_p(T) = \sum_{i=1}^{n} x_{mi} C_{pi}(T)$$ \hspace{1cm} (2)

$$k(T) = \sum_{i=1}^{n} \varepsilon_i k_i(T)$$ \hspace{1cm} (3)

Polynomial expressions for specific heat and thermal conductivity of components are drawn from Singh (1992). Five constituents are considered: fat, protein, water, ash, and air. The volume fraction of each component is given by Equation (4)

$$\varepsilon_i = \frac{\rho_{app} x_{mi}}{\rho_i}$$ \hspace{1cm} (4)

$$\rho_{app}$$ was determined using liquid pycnometry with toluene (Özisik, 1985; Yan et al., 2008).

**Boundary Conditions**

For describing heat transfer in boundary conditions, the equations of heat transfer due to convective stream were applied:

At \(x = 0\),

With conduction:

$$-k(T) \frac{\partial T}{\partial x} = \frac{T - T_w}{\frac{1}{h_m} + \frac{\Delta X_{cap}}{k_{cap}} + \frac{\Delta X_{hs}}{k_{hs}}}$$

with natural convection:

$$-k(T) \frac{\partial T}{\partial x} = \frac{T - T_w}{\frac{1}{h_m} + \frac{\Delta X_{cap}}{k_{cap}} + \frac{1}{h_{hs}}}$$ \hspace{1cm} (5)

At \(x = H\),

$$-k(T) \frac{\partial T}{\partial x} = \frac{T - T_w}{\frac{1}{h_m} + \frac{\Delta X_g}{k_g}}$$ \hspace{1cm} (6)

At \(r = 0\),

$$\frac{\partial T}{\partial r} = 0$$ \hspace{1cm} (7)

At \(r = R\),

$$-k(T) \frac{\partial T}{\partial r} = \frac{T - T_{out}}{h_m} + \frac{\Delta X_g}{k_g}$$ \hspace{1cm} (8)

Where, \(h_m\) (convective heat transfer coefficient at surface) was determined with experiments according to the transient temperature method (Creed and James, 1985) and \(h_{hs}\) (natural convective heat transfer coefficient in headspace) was estimated by the following empirical equations (Özisik, 1985):

$$h_{hs} = \frac{k_a}{\delta}$$ \hspace{1cm} (9)

$$Nu = c(Ra)^n \left( \frac{Hf}{\delta} \right)^m$$ \hspace{1cm} (10)

Temperature was assumed constant and uniform in the product at time zero.

$$T = T_0$$ \hspace{1cm} (11)

**Sterilizing Value (F\(_T\) value) Calculation**

The purpose of this calculation is to arrive at an appropriate process time under a given set of heating conditions to result in a given process lethality, or, alternatively, to estimate the process lethality of a given process. The method used must accurately integrate the lethal effects of the transient temperature response of the food undergoing the thermal processes with respect to the considered microorganism of both public health and spoilage concern. In order to characterize the effect of temperature evolution on microorganism destruction at a given location during pasteurization, a so-called sterilizing value (F\(_T\)) can be calculated:

$$F_T = \int_0^{10^{(T_{ref} - T)}} dz$$ \hspace{1cm} (12)

Where, \(F_T\) represents the duration of the thermal treatment at a constant reference temperature, which would have the same destruction effect as the applied thermal processing on microorganisms. Reference temperatures are 121.1°C (250°F) for heat
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resistant spores to be inactivated in commercial sterilization processes and 82.2°C (180°F) for vegetative cells and organisms of low resistance, which are inactivated in pasteurization processes (Toledo, 2007). Al-Holy et al. (2004) indicated that Listeria innocua is the most heat resistant vegetative microorganism in salmon caviar with \( z_{\text{value}} = 10^\circ\text{F} (5.5^\circ\text{C}) \).

### Numerical Method

A finite difference method was used to solve the heat transfer equations according to Crank-Nicolson’s scheme (Özisik, 1985). It should be noted that the heat transfer equations are coupled by the thermo physical properties that are the function of temperature. At each time step, the values of \( T \) were calculated at each node. After each time step, the thermal properties were estimated according to the new local temperatures. The harmonic means were estimated for \( \kappa_{i+1/2} \) value by the following equations (Patankar, 1980):

\[
\kappa_{i+1/2} = \frac{2k(T_i)k(T_{i+1})}{k(T_i) + k(T_{i+1})}
\]

The procedure outlined above was implemented using a computer program written in MATLAB (version 7.8.0 R2009a).

### Parameters Used during Calculations

The inputs of the mathematical model are: water bath temperature, initial product dimensions, caviar density, component mass fractions, convective heat transfer coefficient at surface of the jar, initial product temperature, number of nodes in the space, and heating and cooling time of pasteurization. Based on these inputs, the model will determine the values of temperature at each node for each time step. The thermo physical properties and operation conditions used in the model are shown in Table 1.

### Table 1. Thermophysical properties and the other inputs used in the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{\text{app}} ) (kg m(^{-3}))</td>
<td>1072.07</td>
</tr>
<tr>
<td>( H ) (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>( h_w ) (W m(^{-2}) K(^{-1}))</td>
<td>200</td>
</tr>
<tr>
<td>( k_s ) (W m(^{-1}) K(^{-1}))</td>
<td>1.05</td>
</tr>
<tr>
<td>( k_{\text{app}} ) (W m(^{-1}) K(^{-1}))</td>
<td>0.19</td>
</tr>
<tr>
<td>( \Delta t ) (s)</td>
<td>2</td>
</tr>
<tr>
<td>( \Delta X_a ) (m)</td>
<td>0.06</td>
</tr>
<tr>
<td>( \Delta X_g ) (m)</td>
<td>0.005</td>
</tr>
<tr>
<td>( \Delta X_{\text{cap}} ) (m)</td>
<td>0.005</td>
</tr>
<tr>
<td>( M )</td>
<td>18</td>
</tr>
<tr>
<td>( N )</td>
<td>14</td>
</tr>
<tr>
<td>( R ) (m)</td>
<td>0.02</td>
</tr>
<tr>
<td>( T_{w,cold} ) (K)</td>
<td>297.65</td>
</tr>
<tr>
<td>( T_{o} ) (K)</td>
<td>299.65</td>
</tr>
<tr>
<td>( T_{\text{ref}} ) (K)</td>
<td>355.35</td>
</tr>
<tr>
<td>( \text{Time} ) (s)</td>
<td>7850</td>
</tr>
<tr>
<td>( x_{\text{ash}} )</td>
<td>0.01</td>
</tr>
<tr>
<td>( x_f )</td>
<td>0.22</td>
</tr>
<tr>
<td>( x_p )</td>
<td>0.20</td>
</tr>
<tr>
<td>( x_w )</td>
<td>0.57</td>
</tr>
<tr>
<td>( z_{\text{value}} ) (K)</td>
<td>5.5</td>
</tr>
</tbody>
</table>

### Experimental Methodology

The experiments were carried out on cylindrical glass jars filled with caviar in triplicate. The processing of caviar was done by a series of stages that included roe extraction, salting, packaging and storage. Caviar samples were obtained from Iran Shilat Co. Chemical analyses of caviar samples were performed according to standard protocols established by the Association of Official Analytical Chemists (934.01, 960.52, 948.22 and 942.05 AOAC Official Methods). The chemical composition of the caviar was 57.08%±0.007 moisture, 19.96%±0.142 proteins, 21.57%±0.577 lipid, and 1.39%±0.057 ash (wet basis). Glass jars were filled manually with 42 g caviar and closed by hand without exhausting operation. Because of the high price of caviar, only 40 percent, instead of 90-94 percent, of the container volume was filled with caviar. Height, internal diameter, and
external diameter of the jars were 100±1, 35, and 40 mm, respectively. Height of caviar in the jar was 40±1 mm. The caviar jars were placed at 20°C for 2 hours before pasteurization. Pasteurization was carried out in a water bath (Memmert, Germany) at 55, 60, 65 and 70°C temperatures for 120, 90, 70, and 50 minutes, respectively, and then with cold water at 20°C. The water bath used was equipped with an automatic temperature controller (±0.1°C). The K-type thermocouples (Omega, Stamford, USA) with 0.3 mm diameter (±0.1°C) were placed in the jar (20±1 mm from the bottom) and water bath, to measure the temperature inside the caviar jar and surrounding water (at 60 seconds intervals). Thermocouples were connected to a temperature data logger (SUPCON, R3100, China) with 15 channels to carry out the measurements. The cylindrical jar containing caviar was immersed in a vertical position in the water bath and the temperature data acquisition was started. After finishing the heating time, the caviar jar was cooled in another incubator with water (24°C). The data were used to validate the developed model. Surface heat transfer coefficient was determined according to the transient temperature method (Creed and James 1985; Mohsenin 1980).

RESULTS AND DISCUSSION

Model Validation

Figure 2 presents the experimental heat penetration curves in caviar jar during pasteurization in water bath at 55, 60, 65, and 70°C. When the caviar sample is placed in the water bath, the surface temperature increases quickly, while the temperature increase in the center is slower. As observed, the heating rate increased with increasing hot water temperature. This was due to the larger temperature gradient between the center and surface of the jar at the higher temperatures.

The model validation was done by comparing the predicted heat penetration curves with the experimental results. Figure 3a shows a typical graph of the comparison between the experimental and predicted temperature evolutions for both heat transfer mechanisms supposed (conduction or natural convection) in headspace. With the conductive heat transfer in headspace, a good agreement was obtained, especially at
the higher temperatures. But, with the natural convective heat transfer, the predicted temperature increase was slightly too fast. This can be explained by the more intensive heat transfer in the case of natural convection. As the predicted temperatures are in better agreement with the experimental results using the conductive heat transfer assumption in headspace, the results obtained with this boundary condition are now analyzed.

Figure 3b presents the regression line between experimental and simulated central temperature. It is clearly seen and proven by statistical analysis that the correlation between experimental and predicted values is very high. The correlation coefficient (r) and the root mean square of errors (RMSE) at four different temperatures were greater than 0.99 and less than 0.61°C, respectively. Also, the maximum difference between experimental and predicted temperatures was 1.15°C. It can be concluded that the assumptions applied to develop the model described the heat transfer mechanisms well during caviar pasteurization.

**Determination of Cold Spot Location**

After validation, the developed model was used to determine the cold spot location of the caviar jar. Cold spot is the location where the temperature increases more slowly than elsewhere. In fact, it is the spot with the lowest F-value. Therefore, determination of this spot is very important to do thermal processing correctly. Figure 3c, Figures 4 and 5 represent the results of the simulation for a caviar jar heated from all sides (at 65°C) in a water bath. Figure 4 shows the temperature evolution at four different points \([a (0,0), b(0, M/2), c (0, M) \) and d \((N, M/2)\)] in the jar during pasteurization. As indicated, temperature in point (b) increases slower than the other points. Figure 5 presents the evolution of predicted temperatures at three radiuses (center, middle and wall) along the vertical axis and at three heights (bottom, middle and top) along the radial axis of the jar at the end of 3,000 seconds heating at 65°C. It could be seen that the temperature increases rapidly at the wall, top, and bottom surfaces of the jar. It can be concluded that the cold spot is a stationary point, and its location is at the radial center between the middle and top of the jar on the vertical axis. Contrary to the other studies, the results of our study show that cold point is shifted on the vertical axis from the middle to the top of the jar.

Headspace volume value of canned foods varies between 6-10% of total volume. In this work, simulations were done for two cases with different headspace volume values (6 and 10%). Figure 6 shows the impact of headspace volume reduction on the cold spot location. This signified that the cold spot location moved slightly towards the middle of the jar on the vertical axis with volume reduction. However, no significant differences were observed between the two cases.

**Estimated Sterilizing Value**

Sterilizing value can be calculated using the predicted temperatures at the cold spot of the caviar jar from Equation (12). Figure 7 shows the evolution of the simulated temperatures at the cold spot during pasteurization at 55 and 65°C.

The comparison of simulated \(F\) values at 55 and 65°C (\(T_{ref}=82.2°C\) and \(z_{value}=5.5°C\)) are presented in Figure 8. As can be seen, the lower the heating temperature, the lower the \(F_{value}\). Table 2 shows the comparison between simulated heating times to obtain an equal sterilization value (\(F_{82.2°C}=0.19\) min) at the different pasteurization temperatures (55, 60, 65, 70, and 74°C). Therefore, in order to achieve the desired \(F\)-values at 65 and 55°C, respectively, 37.63 and 128 minutes are required. This is consistent with the reported values by Johannesson (2006) for pasteurization of the lumpfish caviar.


**Figure 3.** Typical graph of the comparison between the experimental and predicted temperature evolutions for both heat transfer mechanisms assumed (conduction or natural convection) in headspace. (a) Regression line between experimental and simulated central temperature (b) Regression line between experimental and simulated $F$ values (c).
Figure 4. Evolution of temperature at four different points [a (0, 0), b (0, M/2), c (0, M) and d (N, M/2)] in jar.

Figure 5. Evolution of predicted temperatures at three radiiuses (center, middle and wall) along the vertical axis and at three heights (bottom, middle and top) along the radial axis of the jar after 3,000 seconds heating at 65°C.

(Dashed lines are representing wall and bottom of container, respectively).

Figure 6. Impact of headspace volume reduction on the cold spot location.
Figure 7. Evolution of the simulated temperatures at the cold spot during pasteurization time at the different heating temperatures (Water temperature: 55 and 65°C).

Figure 8. Comparison of simulated F values with $T_{\text{ref}} = 82.2°C$ and $z_{\text{value}} = 5.5°C$ (Water temperature: 55 and 65°C).

Table 2. The simulated heating time to achieve $F_{82.2°C}$ = 0.19 min and cold point temperature at the end of heating ($T_{\text{ref}}= 82.2°C$ and $z_{\text{value}}= 5.5°C$).

<table>
<thead>
<tr>
<th>Heating Temperature (°C)</th>
<th>55</th>
<th>60.46</th>
<th>37.63</th>
<th>27.58</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent Heating Time (Min)</td>
<td>128</td>
<td>60.46</td>
<td>37.63</td>
<td>27.58</td>
<td>23</td>
</tr>
<tr>
<td>Cold point temperature (°C)</td>
<td>54.99</td>
<td>59.78</td>
<td>63.10</td>
<td>64.69</td>
<td>65.25</td>
</tr>
</tbody>
</table>
CONCLUSIONS

A numerical model was developed to simulate two-dimensional heat transfer in homogenous finite cylinder (caviar jar) to predict the local temperature and $F_{\text{value}}$ during pasteurization. The model accommodates the effects of headspace volume value and temperature dependent variables such as specific heat and thermal conductivity. The model was validated by comparison of the experimental temperature profiles during pasteurization of caviar with the predicted values. The main results that can be withdrawn from this study are the following: (i) the cold spot location is at the radial center between the middle and top of the jar on the vertical axis; (ii) with headspace volume reduction from 10 to 6%, the cold spot location moved slightly towards middle of the jar on the vertical axis; and (iii) the simulated temperatures, $F_{\text{value}}$ and heating times required to obtain a desired F-value show a good agreement with the experimental results and the reported values in the literature. It can be concluded that the developed model is useful to describe the heat transfer phenomenon during caviar pasteurization. Work will continue on thermal process optimization of the caviar by application of the developed model.

Nomenclature

c  Constant
$C_p$  Specific heat, J kg$^{-1}$K$^{-1}$
$F$  Equivalent Heating Time at reference temperature, min
$H$  Jar height, m
$H_f$  Height of fluid layer in headspace, m
$h_{ha}$  Natural convective heat transfer coefficient in headspace, W m$^{-2}$K$^{-1}$
$h_m$  Average convective heat transfer coefficient at surface, W m$^{-2}$K$^{-1}$
k  Thermal conductivity, W m$^{-1}$K$^{-1}$
m  Constant
$M$  Number of axial nodes

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مدل‌سازی ریاضی انتقال حرارت و برآورده‌رژش استریل کندگی دما طی پاستوریزاسیون خاویار

م. خاکی‌آب، ح. شاهدی، ن. همدانی، ع. حجازی، ع. مظلی

چکیده

در این مطالعه، یک مدل عددی برای بررسی نحوه انتقال حرارت و برآورده‌رژش استریل کندگی دما طی پاستوریزاسیون خاویار توسعه داده شد. در این راستا روش تفاضل محدود برای حل عددی روابط انتقال حرارت بر اساس شیمی کرائکل پلی‌اسپرون مورد استفاده قرار گرفت. سپس مدل توسط مقایسه بر روی الیاف های دما به دست آمده از پاستوریزاسیون خاویار با مقادیر پیش بینی شده توسط مدل، معترضاتی نشد. در نتیجه نتایج نشان دادند که مدل توسعه داده شده برای شبیه سازی حرارتی خاویار به شکل موفقیت آمیز مورد استفاده قرار گرفته است.