

## Factor $H$ for the Calculation of Head Loss and Sizing of Dual-diameter Laterals

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### ABSTRACT

Factor  $H$  is introduced for direct head loss calculation and sizing in dual-diameter laterals of sprinkler and trickle irrigation systems when the Hazen-Williams equation is to be applied. Application of this factor prevents trial and error and uses fewer head-loss equations in calculating the total friction loss and designing dual-diameter laterals. The proposed factor is a function of total number of outlets on the lateral, number of outlets on the smaller-diameter section of the pipe and ratio of smaller diameter to the larger diameter pipe. By solving two practical examples, the advantage of this factor over other approaches was shown. It was also demonstrated that design of lateral sizing by the given equations is simpler, easier and more accurate than the previous methods.

**Keywords:** Correction factor, Friction loss, Irrigation systems, Tapered lateral pipe.

### INTRODUCTION

In surface, sprinkle, and trickle irrigation systems, lateral pipelines with multiple outlets are used for supplying water to the field. In solid set, periodic move, or linear move sprinkle systems, the outlets are uniformly spaced along the laterals and are assumed to have uniform discharge. These characteristics also apply to most trickle irrigation systems and gated pipes for surface irrigation (Scaloppi and Allen, 1993). In a lateral with multiple outlets, the discharge through the pipeline decreases along the length of the pipe. As a result, the head loss caused by friction will be less than that in a similar pipeline without outlets. Computation of the head loss caused by friction requires a stepwise manner. According to this procedure, friction losses are calculated individually for each hydraulic

lateral section (starting from the distal end of the lateral) by a stepwise procedure assuming uniform and discrete outflow from outlets.

Several researchers have developed the concept of a reduction factor for closed-end multiple-outlet laterals, beginning with Dupuit (1865) for continuous outflow water pipes. Christiansen (1942) developed the widely used factor  $F$  to avoid the cumbersome stepwise analysis required to calculate head loss in pipelines with outlets. This factor can be analytically calculated using the following equation (DeTar, 1982):

$$F = \frac{1}{m+1} + \frac{1}{2N} + \frac{m}{12N^2} \quad (1)$$

Where,  $N$  is total number of outlets and  $m$  is exponent of velocity or discharge term in friction formula used. Factor  $F$  is dimensionless and is a function of the friction formula used (Darcy-Weisbach or Hazen-

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Williams) and the number of outlets along the pipeline. In many field layouts, however, the first outlet on the lateral is not located a full spacing from the inlet. Jensen and Fratini (1957) addressed this issue by developing an adjusted factor  $F_a$ . This factor permits calculating the head loss caused by friction in laterals with multiple outlets, with the first outlet at one-half of the full spacing from the inlet. Chu (1978) modified the adjusted factor  $F_a$  of Jensen and Fratini (1957) and suggested that this modified factor  $F_a$  could be considered constant for five or more outlets. Cuenca (1989) presented a solution to the Jensen and Fratini (1957) while DeTar (1982), Valiantzas (2002a) and Sadeghi et al. (2010) arrived at results similar to Christiansen (1942). Scaloppi (1988) derived an expression for the adjusted factor  $F_a$ . This expression allows the correction factor to be calculated for a lateral with multiple outlets and the first outlet at any fraction of spacing from the inlet.

Factor  $F$  and adjusted factor  $F_a$  provide very convenient tools for calculating head loss caused by friction in a pipeline with multiple outlets. However, these factors can only be used directly for pipelines with a single diameter. When dealing with multiple-diameter (tapered) laterals, the friction loss calculation requires using the factor  $F$  in an indirect method (Hart, 1975; Keller and Bliesner, 1990) as:

$$H_{f_{ac}} = H_{f_a} - H_{f_b} + H_{f_c} \quad (2)$$

where  $H_{f_{ac}}$  is actual head loss in the lateral,  $H_{f_a}$  is head loss in smaller diameter section  $D_2$ , in which  $H_{f_a} = f(D_2, L_2, F_2, Q_2)$ ,  $H_{f_b}$  is head loss in larger diameter section  $D_1$ , in which  $H_{f_b} = f(D_1, L_2, F_2, Q_2)$ ,  $H_{f_c}$  is head loss for the section with diameter  $D_1$  using the flow rate for the entire lateral, in which  $H_{f_c} = f(D_1, L, F, Q_{in})$ ,  $D_1$  is diameter of the larger-size section,  $D_2$  is diameter of the smaller-size section,  $L_1$  is

length of the larger diameter section,  $L_2$  is the length of the smaller diameter section,  $L$  is the total length of the lateral,  $F_2$  is Christiansen friction factor for the smaller diameter section,  $Q_{in}$  is flow rate entering the lateral, and  $D_2$  is flow rate in the smaller diameter section. The  $H_{f_i}$ ,  $D_i$  and  $L_i$  parameters have units of length and  $Q_i$  have units of volume/time. Clearly, using this method requires calculation of three head losses. Also, round-off errors are going to be a problem in the final head loss value of the lateral.

In order to simplify the above procedure, Anwar (1999a) developed a  $G$  factor that permits calculating the head loss caused by friction in the pipe with multiple outlets and outflow at the downstream end. Factor  $G$  can be applied to each reach within a tapered pipe. However, its application is limited in that it assumes that the first outlet is one full spacing from the pipeline inlet. Anwar (1999b) completed his previous work by developing factor  $G_a$  for pipes with multiple outlets and outflow at the downstream when the first outlet can be situated at any distance from the inlet. Anwar (1999b) assumed the following hypotheses in developing the  $G$  factor:

- The outlets are equally spaced and of uniform discharge.
- The friction factor remains constant along the pipe length.
- The velocity head can be neglected.
- The increase in pressure past each outlet, caused by reduction of flow, is equal to the head loss caused by turbulence associated with each outlet (Pair et al., 1973).
- Head loss at the junction of two sizes in the pipe is ignored.

The analytical form for the Anwar's  $G$  factor was recently found by Sadeghi and Peters (2011) using the continuous approach in which an infinite number of outlets are considered along the lateral:

$$G = \frac{[N(1+r)+0.5]^{m+1} - (Nr+0.5)^{m+1}}{(1+r)^m(m+1)N^{m+1}} \quad (3)$$

Where,  $r$  is the ratio of outflow discharge to total pipe discharge (see Anwar, 1999a, b).

the Hazen-Williams equation is to be applied.

### Sizing Tapered Laterals

As multiple-diameter laterals reduce the total cost in sprinkle and trickle systems, their design is very important and therefore the length of each section of such pipes should be determined carefully. Up to now, the best method for this is trial and error, which is time-consuming. Recently, Yitayew (2009) presented a direct analytical approach that gives a solution for a combination of diameters of different lengths. This approach gives accurate results when several outlets exist on the pipe, but again a trial and error method is needed for small number of outlets (Example 3 of Yitayew, 2009).

The present paper presents a new factor, called factor  $H$ , considering the same assumptions of Anwar (1999a, b). It helps fast, accurate and direct calculation of total head loss for multiple-diameter lateral pipes. As follows, first, the theoretical approach is presented and then, application of this new factor is compared with some researchers' procedures in calculation of friction head loss and design of dual-size laterals. The limitation associated with the new proposed factor is that because the friction factor is considered constant along the total length of the lateral, its application is only valid when

### Method of Analysis

#### a) First Case: Discrete Outflow in Outlets

Consider a dual-diameter lateral pipe in a sprinkler or trickle irrigation system (Figure 1). The diameter of the first section (larger size) of the lateral is  $D_1$ , its length is  $L_1$ , and number of outlets is  $N_1$ . Similarly,  $D_2, L_2$ , and  $N_2$  are diameter, length, and number of outlets of the second section (smaller size). Outlets on the lateral are assumed to have equal discharge of  $q$ , and the spacing between them is assumed to be  $l$ .

The general form of equation for computing friction loss in a pipe (without any outlet) can be shown as:

$$H_f = TD^n LQ_{in}^m \quad (4)$$

Where,  $H_f$  is total friction head loss for a pipe without any outlet,  $T$  is a coefficient based on the pipe material and the friction formula used,  $Q_{in}$  is discharge flowing into the lateral,  $L$  is total length of the pipe, and  $n$  and  $m$  are exponents which depend on the friction formula used.

By applying the above equation, the

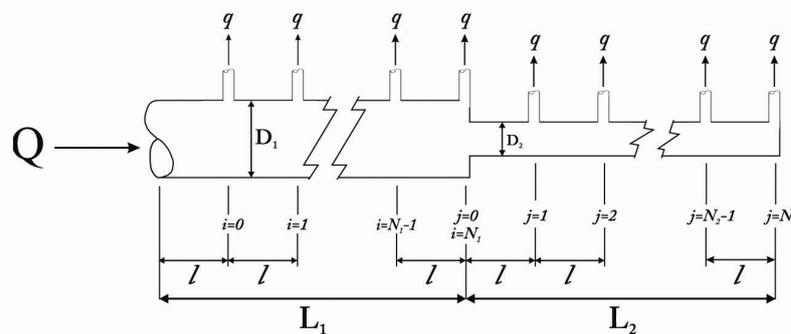


Figure 1. Scheme of a dual-diameter lateral pipeline in an irrigation system.



friction head loss for each segment of the lateral can be written as:

$$\begin{aligned}
 h_{f_1} &= TD_1^n Q_{in}^m \\
 h_{f_2} &= TD_1^n (Q_{in} - q)^m \\
 &\dots \\
 &\dots \\
 h_{f_{N_1}} &= TD_1^n l (Q_{in} - (N_1 - 1)q)^m \\
 h_{f_{N_1+1}} &= TD_2^n l (Q_{in} - N_1 q)^m \\
 h_{f_{N_1+2}} &= TD_2^n l (Q_{in} - N_1 q - q)^m \\
 &\dots \\
 &\dots \\
 h_{f_N} &= TD_2^n l (Q_{in} - N_1 q - (N_2 - 1)q)^m
 \end{aligned} \tag{5}$$

Where,  $h_{f_1}$ ,  $h_{f_2}$ ,  $h_{f_{N_1}}$ ,  $h_{f_{N_1+1}}$ ,  $h_{f_{N_1+2}}$  and  $h_{f_N}$  are head loss in the first, second,  $N_1$ th,  $N_1+1$ th,  $N_1+2$ th and  $N$ th part of the lateral. Then, the actual head loss in the lateral is:

Taking into account the terms of Eq. (5), considering  $N_1 = N - N_2$ , and  $Q_{in} = Nq$ , Equation (6) simplifies to Equation (7):

Now consider a hypothetical lateral pipe that its length and diameter are  $L$  and  $D_1$ , respectively but does not have any outlets. Total head loss of this pipe is calculated from Equation (4) as:

$$H_{f_{HYPO}} = TLQ_{in}^m D_1^n \tag{8}$$

The question is this: what factor should  $H_{f_{HYPO}}$  be multiplied by to get the  $H_{f_{ac}}$ ?

$$H_{f_{ac}} = H_{f_{HYPO}} \times H \tag{9}$$

In order to find the general equation

$$H_{f_{ac}} = h_{f_1} + h_{f_2} + \dots + h_{f_{N_1}} + h_{f_{N_1+1}} + h_{f_{N_1+2}} + \dots + h_{f_N} \tag{6}$$

$$H_{f_{ac}} = Tlq^m D_1^n \left[ \sum_{i=1}^{N-N_2} (N-i-1)^m + \left\{ \left( \frac{D_2}{D_1} \right)^n \times \sum_{j=0}^{N_2-1} (N_2-j)^m \right\} \right] \tag{7}$$

$$H_f = \frac{H_{f_{02}}}{(m+1)} \left[ \left( \frac{L_2}{L} + \frac{1}{2N} \right)^{m+1} - \left( \frac{1}{2N} \right)^{m+1} \right] + \frac{H_{f_{01}}}{(m+1)} \left[ \left( 1 + \frac{1}{2N} \right)^{m+1} - \left( \frac{L_2}{L} + \frac{1}{2N} \right)^{m+1} \right] \tag{12}$$

expressing the  $H$  factor, let:

$$\alpha = \frac{D_2}{D_1} \tag{10}$$

Since practical lateral pipe sizes for application in trickle and sprinkler irrigation systems are 12, 16, 20, 25, 32, 40, 50, 63, 75, 90 and 110 mm,  $\alpha$  in Equation (10) would only accept some specific values

as  $\frac{12}{16}, \frac{16}{20}, \dots, \frac{90}{110}$ . Substitution of Equations (7) and (8) in Equation (9) and rearranging for  $H$  yields:

$$H = \frac{\sum_{i=1}^{N-N_2} (N-i-1)^m + \left\{ \alpha^n \sum_{j=0}^{N_2-1} (N_2-j)^m \right\}}{N^{m+1}} \tag{11}$$

To read  $H$  directly for different values of  $N$  and  $N_2$ , tables could be prepared for various  $\alpha$  values. Also, Equation (11) could be extended by Taylor series and its analytical form be written (see for example: Anwar, 1999a). However, an analytical solution can also be found using the continuous outflow variation along the lateral. This concept will be dealt with in the next section.

### b) Second Case: Continuous Outflow in Outlets

Analytical determination of total energy line is based on the assumption that the outflow varies continuously spatially along the lateral, which has an infinite number of outlets. In line with this concept, the total friction head loss for the given lateral in Figure 1 according to the Valiantzas's continuous outflow approximation is

Equation (12), (Equation (13) of Valiantzas (2002b) with an adverse definition of the used indices here):

Where:

$$H_{f_{01}} = TD_1^n LQ_{in}^m \quad (13a)$$

And:

$$H_{f_{02}} = TD_2^n LQ_{in}^m \quad (13b)$$

Substituting Equations (13a) and (13b) in Equation (12), taking into account Equation (10) and also  $L_2/L = N_2/N$ , gives Equation (14):

Finally, substituting Equation (8) in Equation (9) and equating the result with Equation (14) gives the analytical form of the  $H$  factor as Equation (15).

Figure 2 illustrates the variation of  $H$  from Equation (15) compared with the  $H$  factor obtained by a stepwise analysis (Equation (11)) for  $m = 1.852$ ,  $N = 15, 40$ , and  $100$  and  $\alpha = 0.67$  and  $0.75$ . As a remarkable result, the  $H$  values obtained from Equation (15) are in excellent agreement with the stepwise method.

### c) Designing Tapered Laterals

A properly designed lateral will operate within the design constraints as long as the actual head loss due to friction along it is less than the allowable head loss (Cuenca, 1989). Hence, the following procedure is taken here to design a dual-size lateral:

1- Using Christiansen formula ( $H_{f_c} = F \times H_f^n$ ), total allowable friction loss,  $N$  outlets, total discharge of  $Q_{in}$ , and total length of  $L$ , the diameter of the lateral pipe is estimated. This fictitious diameter should be in the range of available diameters in the market. Therefore,  $D_1$  and  $D_2$ , and consequently  $\alpha$ , are determined at this

$$H_f = \frac{TLQ_{in}^m D_1^n}{m+1} \left[ \alpha^n \left\{ \left( \frac{N_2}{N} + \frac{1}{2N} \right)^{m+1} - \left( \frac{1}{2N} \right)^{m+1} \right\} + \left\{ \left( 1 + \frac{1}{2N} \right)^{m+1} - \left( \frac{N_2}{N} + \frac{1}{2N} \right)^{m+1} \right\} \right] \quad (14)$$

stage.

2- Based on the allowable head loss, the allowable  $H$  factor, called  $H_{allow}$  is calculated from the following equation:

$$H_{allow} = \frac{H_{f_{allow}}}{TLQ_{in}^m D_1^n} \quad (16)$$

Where,  $H_{f_{allow}}$  is the allowable head loss.

3- The value of  $N_2$  and consequently  $L_2$  and  $L_1$  are found by substituting  $H_{allow}$  for  $H$  in Equation (15) and solving for  $N_2$ . In other words,  $N_2$  can be calculated using the explicit equation (17).

Where, "Int" means that the integer part of Equation (17) is taken as the  $N_2$  value.

### d) Practical Application

**Example 1-** Calculate the friction head loss in a sprinkler lateral that is 288 m long. Sprinklers are installed at 12 m intervals. The first 144 m (starting at the inlet) of the lateral has an internal diameter of 101.6 mm (4 inches) and the next 144 m of the lateral has an internal diameter of 76.2 mm (3 inches). There are 24 outlets on the lateral, each discharging  $0.5 \text{ L s}^{-1}$  and the first sprinkler is 12 m from the inlet. Use Hazen-Williams equation for the calculation of the head loss ( $C_{HW} = 130$ ).

**Solution:** This problem is solved by two different well-known methods to show that they are either lengthy or have some weaknesses.

### Hart (1975) Method

For this example, we have  $D_1 = 0.1016 \text{ m}$ ,  $L = 288 \text{ m}$ ,  $D_2 = 0.0762 \text{ m}$ ,  $L_2 = 144 \text{ m}$ ,

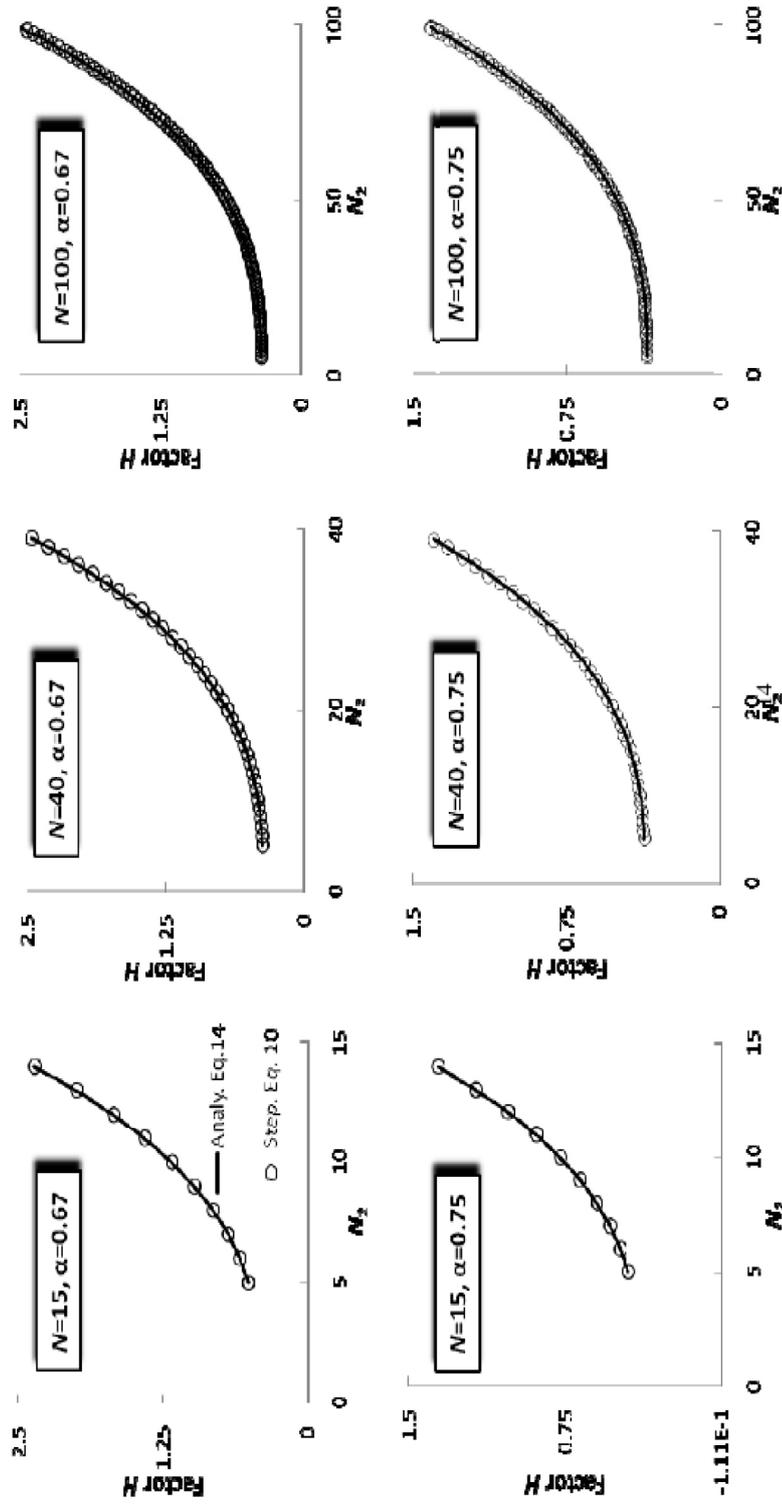


Figure 2. Variations of the proposed  $H$  factor calculated from the analytical Equation (14) and comparison with the stepwise Equation (10) for  $m = 1.852$ ,  $N = 15, 40$ , and  $100$  and  $\alpha = 0.67$  and  $0.75$ .

$$H = \frac{\alpha^n \times \left\{ \left( \frac{N_2}{N} + \frac{1}{2N} \right)^{m+1} - \left( \frac{1}{2N} \right)^{m+1} \right\} + \left\{ \left( 1 + \frac{1}{2N} \right)^{m+1} - \left( \frac{N_2}{N} + \frac{1}{2N} \right)^{m+1} \right\}}{m+1} \quad (15)$$

$$N_2 = \text{Int} \left[ N \left[ \frac{\left( (m+1)H_{allow} - \left( 1 + \frac{1}{2N} \right)^{m+1} + \frac{\alpha^n}{(2N)^{m+1}} \right)^{\frac{1}{m+1}}}{\alpha^n - 1} \right] - 0.5 \right] \quad (17)$$

$N_2=12$ ,  $Q_{in}= 24 \times 0.5= 12 \text{ L s}^{-1}$ , and  $Q_2= 12 \times 0.5= 6 \text{ L s}^{-1}$ . From Equation (1),  $F_2= 0.3934$ . Also,  $F_N= 0.3717$ . Note that  $F_N$  and  $F_2$  are respectively the Christiansen  $F$  factor the total  $N$  outlets and also the total outlets located on the downstream reach of the lateral.

For multi-outlet laterals, the Hazen-Williams equation in metric system (SI) is written as follows:

$$H_f = 10.672 \times F_C \times \left( \frac{Q_{in}}{C_{HW}} \right)^{1.852} \times L \times D^{-4.871} \quad (18)$$

Therefore, going through Equations (17) and (2), we have:

$$H_{f_a} = 10.672 \times \left( \frac{0.006}{130} \right)^{1.852} \times (0.0762)^{-4.871} \times 0.3934 \times 144 = 1.576 \text{ m}$$

$$H_{f_b} = 10.672 \times \left( \frac{0.006}{130} \right)^{1.852} \times (0.1016)^{-4.871} \times 0.3934 \times 144 = 0.388 \text{ m}$$

$$H_{f_c} = 10.672 \times \left( \frac{0.012}{130} \right)^{1.852} \times (0.1016)^{-4.871} \times 0.3717 \times 288 = 2.648 \text{ m}$$

And finally,  $H_{f_{ac}} = H_{f_a} - H_{f_b} + H_{f_c} = 3.836 \text{ m}$ .

#### Anwar (1999a) Method

Using Hazen-Williams formula for the second section of the lateral gives:

$$H_f = 10.672 \times \left( \frac{0.006}{130} \right)^{1.852} \times (0.0762)^{-4.871} \times 144 = 4.006 \text{ m}$$

The ratio of outflow discharge to total discharge through the outlets along the pipe is denoted by Anwar (1999a) as:

$$r = \frac{Q_o}{Nq} \quad (19)$$

Where,  $q$  is the outlet discharge and  $Q_o$  is the outflow past the last sprinkler. Here, since there is no outflow past the last sprinkler in section 2,  $Q_o = 0$  and  $r = 0$ .

Factor  $G$  for  $r=0$  and  $N=12$  (from Equation (3)) is 0.394. Then:

$$H_{f_2} = G \times H_f = 0.394 \times 4.006 = 1.578 \text{ m}$$

Where,  $H_{f_2}$  is total friction loss for the second section of the lateral.



On the other hand, the Hazen-Williams equation for the first section of the lateral yields:

$$H_f = 10.672 \times \left( \frac{0.012}{130} \right)^{1.852} \times (0.1016)^{-4.871} \times 144 = 3.562 \text{ m}$$

For the first section of the lateral, the outflow at downstream end is the discharge into the second segment ( $Q_0 = 6 \text{ L/s}$ ). Discharge of the first section  $Q_1 = 12 \times 0.5 = 6 \text{ L/s}$ . Hence, the value of  $r$  for the first segment is calculated from Eq. (18) as:

$$r = \frac{6}{12 \times 0.5} = 1$$

Again, from Sadeghi and Peters (2011), the value of  $G$  for  $r = 1$  and  $N = 12$  is 0.635.

Therefore,  
 $H_{f1} = G \times H_f = 0.635 \times 3.562 = 2.262 \text{ m}.$

Finally,

$$H_{f_{ac}} = H_{f_1} + H_{f_2} = 2.262 + 1.578 = 3.84 \text{ m}.$$

### The Method of This Paper

From Equation (9),  $\alpha = D_2 / D_1 = 0.75$ . Using Equation (14) for  $N = 24$  and  $N_2 = 12$  yields  $H = 0.5388$ . Then, from Equation (7):

$$H_{f_{HYPO}} = 10.672 \times \left( \frac{0.012}{130} \right)^{1.852} \times (0.1016)^{-4.871} \times 288 = 7.123 \text{ m}$$

and finally from Equation (8), we obtain:

$$H_{f_{ac}} = H_{HYPO} \times H = 3.838 \text{ m}.$$

The same problem can be solved in a stepwise manner, starting from the pipe inlet and proceeding toward downstream of the pipe. The results of these computations are shown in Table 1. As is shown in this table,

**Table 1.** Stepwise solution to Example 1.

Segment	$L$ (m)	$Q_{in}$ ( $\text{L s}^{-1}$ )	$D$ (m)	$h_f$ (m)	$\sum h_f$ (m)
1	12	12.0	0.1016	0.2968	0.2968
2	12	11.5	0.1016	0.2743	0.5711
3	12	11.0	0.1016	0.2526	0.8237
4	12	10.5	0.1016	0.2318	1.0555
5	12	10.0	0.1016	0.2118	1.2673
6	12	9.5	0.1016	0.1926	1.4598
7	12	9.0	0.1016	0.1742	1.6340
8	12	8.5	0.1016	0.1567	1.7908
9	12	8.0	0.1016	0.1401	1.9308
10	12	7.5	0.1016	0.1243	2.0551
11	12	7.0	0.1016	0.1094	2.1645
12	12	6.5	0.1016	0.0954	2.2599
13	12	6.0	0.0762	0.3338	2.5937
14	12	5.5	0.0762	0.2842	2.8778
15	12	5.0	0.0762	0.2382	3.1160
16	12	4.5	0.0762	0.1960	3.3120
17	12	4.0	0.0762	0.1575	3.4695
18	12	3.5	0.0762	0.1230	3.5926
19	12	3.0	0.0762	0.0925	3.6850
20	12	2.5	0.0762	0.0660	3.7510
21	12	2.0	0.0762	0.0436	3.7946
22	12	1.5	0.0762	0.0256	3.8203
23	12	1.0	0.0762	0.0121	3.8324
24	12	0.5	0.0762	0.0033	3.8357

$$L_2 = \left[ \left( \frac{H_{f_{allow}} l^{1.852} C_{HW}^{1.852}}{Kq^{1.852} F} - \frac{L^{2.852}}{D_1^{4.852}} \right) \left( \frac{1}{D_2^{4.852}} - \frac{1}{D_1^{4.852}} \right) \right]^{-1} \frac{1}{2.852} \quad (20)$$

total head loss caused by friction using a stepwise analysis is 3.836 m, which agrees well with 3.838 m of calculations.

**Example 2-** A sprinkler lateral is 403 m long with sprinklers 13 m apart on the lateral. The flow rate from each sprinkler is  $0.352 \text{ L s}^{-1}$ . With the allowable head loss set at 10 m, design the lateral. Assume again that the Hazen-Williams coefficient ( $C_{HW}$ ) is 130.

### Yitayew (2009) Solution

When the Hazen-Williams equation is to be used, Yitayew (2009) proposed the following equation to compute the length of the second reach for a specific allowable head loss: Equation (20)

Where,  $L_2$  is the length of the second reach of lateral,  $F$  is Christiansen's friction factor for  $N$  outlets, and  $K$  is a coefficient which only depends on the friction formula used ( $K = 1.22 \times 10^{10}$  for Hazen-Williams equation).

For the given length and spacing, total number of sprinklers will be 31. Then from Equation (1), we obtain  $F = 0.3669$ . The flow rate corresponding to this number of sprinklers is  $Q_{in} = 10.912 \text{ L/s}$ . Substituting  $Q_{in} = 10.912 \text{ L/s}$ ,  $L = 403 \text{ m}$ ,  $H_f = 10 \text{ m}$ , and  $C_{HW} = 130$  in Equation (3), then multiplying the result by  $F$ , we get a diameter  $D = 79.8 \text{ mm}$ . The two standard diameters that meet the above recommendation are  $D_1 = 101.6 \text{ mm}$  and  $D_2 = 76.2 \text{ mm}$ . Therefore, the length of the second segment of the lateral can be determined using Equation (19):

Using the trial-and-error method for  $D_2 = 76.2 \text{ mm}$ ,  $L_2$  is obtained to be equal to 351 m,  $N_2 = 27$ ,  $Q_2 = 9.504 \text{ L/s}$ ,  $F_2 = 0.3694$ ,  $D_1 = 101.6 \text{ mm}$ ,  $L_1 = 52 \text{ m}$ ,  $N_1 = 4$ ,  $L = 403 \text{ m}$ ,  $N = 31$ ,  $Q_{in} = 10.912 \text{ L/s}$ , and  $F = 0.3669$ .

For Hazen-Williams equation,  $H_{f_a}$ ,  $H_{f_b}$  and  $H_{f_c}$  can be derived from the following equations:

$$H_{f_a} = 10.672 \times \left( \frac{0.009504}{130} \right)^{1.852} \times (0.0762)^{-4.871} \times 0.3694 \times 351 = 8.454 \text{ m}$$

$$H_{f_b} = 10.672 \times \left( \frac{0.009504}{130} \right)^{1.852} \times (0.1016)^{-4.871} \times 0.3694 \times 351 = 2.082 \text{ m}$$

$$H_{f_c} = 10.672 \times \left( \frac{0.010912}{130} \right)^{1.852} \times (0.1016)^{-4.871} \times 0.3669 \times 403 = 3.067 \text{ m}$$

Hence, from Equation (2) we obtain:

$$H_{f_{ac}} = 8.454 - 2.082 + 3.067 = 9.439 \text{ m.}$$

Solving Eq. (19) for  $H_{f_{allow}} = H_{f_{ac}} = 9.439 \text{ m}$ , we get  $L_2 = 345.84 \text{ m}$ .

### The Method of This Paper

As it was shown in the previous solution method, the two diameters were  $D_1 = 101.6 \text{ mm}$  and  $D_2 = 76.2 \text{ mm}$ . Therefore, we

$$L_2 = \left[ \left( \frac{10 \times 13^{1.852} \times 130^{1.852}}{1.22 \times 10^{10} \times 0.352^{1.852} \times 0.3669} - \frac{403^{2.852}}{101.6^{4.871}} \right) \left( \frac{1}{76.2^{4.871}} - \frac{1}{101.6^{4.871}} \right) \right]^{-1} \frac{1}{2.852} = 361.92 \text{ m}$$



have  $\alpha = D_2 / D_1 = 0.75$ . Then, from Eq. (16), we have:

$$H_{allow} = \frac{10}{10.672 \times \left( \frac{0.010912}{130} \right)^{1.852} \times 403 \times (0.1016)^{-4.871}} = 1.1963$$

Now, using Equation (16) for  $N = 31$ ,  $\alpha = 0.75$ ,  $m = 1.852$  and  $H_{allow} = 1.1963$ , we obtain  $N_2 = [27.822] = 27$ . Finally,  $L_2$  and  $L_1$  are calculated as  $L_2 = 27 \times 13 = 351$  m and  $L_1 = 403 - 351 = 52$  m as was calculated by the accurate stepwise solution.

## CONCLUSIONS

Calculation of total head loss due to friction in tapered-lateral pipelines of sprinkle and trickle irrigation systems is performed mainly by lengthy or trial-and-error methods. A new factor is proposed and the analytical formula is given for the calculation of total head loss in dual-size laterals. The suggested friction correction factor is to be calculated using the total number of outlets on the lateral, number of outlets on the smaller-diameter section of the pipe, ratio of smaller diameter to the larger diameter, and exponent of velocity in the friction formula used. By solving two practical examples, the advantage of this factor over other approaches was illustrated. It was also demonstrated that design of lateral sizing by the given equations is simpler, easier and more accurate than the previous methods.

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### ضریب تصحیح $H$ برای محاسبه افت اصطکاک و طراحی لوله‌های جانبی دوقطری

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#### چکیده

ضریب تصحیح  $H$  برای محاسبه افت کل اصطکاک و همچنین طراحی لوله‌های جانبی دوقطری مورد کاربرد در سیستم‌های آبیاری بارانی و قطره‌ای، زمانی که استفاده از معادله هیزن-ویلیامز مد نظر است، پیشنهاد گردید. استفاده از این ضریب نه تنها مهندسين را از انجام عملیات سعی و خطا بی‌نیاز می‌کند، بلکه تعداد معادلات لازم جهت محاسبه افت اصطکاک و همچنین طراحی لوله‌های فرعی دوقطری را کاهش می‌دهد. ضریب تصحیح پیشنهادی تابعی از تعداد کل خروجی‌ها، تعداد خروجی‌های قرار گرفته روی قسمتی از لوله جانبی که قطر کمتری دارد و همچنین نسبت قطر کوچکتر به قطر بزرگتر می‌باشد. با حل نمودن دو مثال عددی، مزیت استفاده از ضریب  $H$  در محاسبه افت اصطکاک نسبت به سایر روش‌های پیشنهاد شده تاکنون به اثبات رسید. همچنین نشان داده شد که طراحی لوله‌های فرعی دوقطری با استفاده از ضریب پیشنهاد شده آسانتر و دقیق‌تر از روش‌های پیشین می‌باشد.