Irrigation Planning with Fuzzy Parameters: An Interactive Approach

D. G. Regulwar\textsuperscript{1*}, and J. B. Gurav\textsuperscript{2}

ABSTRACT

Decisions relating to most irrigation-planning problems need to be made in the face of hydrologic uncertainties, which make the irrigation-planning problem more complex. The uncertainties can be tackled by formulating the problem as Fuzzy Linear Programming (FLP). In the present study, Single Objective Fuzzy Linear Programming (SOFLP) irrigation planning model was formulated for deriving the optimal cropping pattern plan with the objective of minimization of cost of cultivation and maximization of net benefits for the case study of Jayakwadi Project Stage-I in Godavari River sub-basin in Maharashtra State, India. The objective function coefficients, technological coefficients, and stipulations/resources under consideration were taken as triangular fuzzy numbers. The interactive approach was used to solve SOFLP model by involving the Decision Maker (DM) in all phases of decision-making process. The SOFLP model gave better results at highest degree of the membership value by keeping balance between feasibility degree of constraints and satisfaction degree of objectives. The minimized cost of cultivation and maximized net benefits for irrigation planning for the SOFLP model proposed, was found at greatest membership degree of 0.406 and 0.331, respectively, with the consideration of balance between the feasibility degree of constraints and satisfaction degree of goal. The DM can be involved in all phases of decision process, which is very essential in real world problems of irrigation planning where the data/information is vague or uncertain.

Keywords: Decision maker, Single objective fuzzy linear programming, Uncertainty.

INTRODUCTION

Linear programming model developed for irrigation planning based on real world complexities considers a number of parameters whose values are given by experts, who are required, in the case of the conventional approach, to place an exact value to these parameters. Many times, the exact values suggested are based on statistical analysis of the past data and its stability is doubtful. Therefore, decision-maker (DM) represents these parameters in an uncertain way or by means of linguistic variables. This can be treated as the source of the fuzziness in the data. Such data represented by experts are considered as fuzzy in nature. The irrigation planning is dependent on many factors such as weather, climate, temperature, rainfall, marketing, and resource availability, which are not easily quantified and often are not fully controllable. These factors are the common sources of uncertainty. In actual planning practice/exercise, the input data and other parameters such as demand, resources, cost, and objective functions are also imprecise (fuzzy) because some information are incomplete or unobtainable. Since uncertainty plays an important role in any irrigation planning, a model with multiple

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objectives that takes into account uncertainty should be used. Conventional mathematical programming schemes lucidly cannot handle all these issues.

Anand Raj and Nagesh Kumar (1998, 1999) developed and employed the new method to rank the fuzzy alternatives along with their total utility value. The method is simple and easy to utilize and is based on the concept of maximizing and minimizing set. The method is applied to Krishna River basin planning and development alternatives under multi-criterion environment. The method proposed by Anand Raj and Nagesh Kumar (1999) can overcome one or more drawbacks of various methods of ranking reported in the literature. This method has the advantages of being intuitive in nature, computationally simple and easy to understand; also, it allows the involvement of more than one expert and helps in ranking of alternatives with fuzzy weights. The method does not consider the existence of some fuzzy relationship or other functional relationships across alternatives, but it avoids crisp ranking from fuzzy data. Raju and Nagesh Kumar (2000a) developed Linear Programming (LP) irrigation planning model for evaluation of irrigation development strategy for the case study of Sri Ram Sagar project, in Andhra Pradesh of India. Uncertainty in inflows arising out of the uncertainty in the rainfall is tackled through chance constrained (stochastic) programming. Raju and Nagesh Kumar (2000b) presented the Fuzzy Linear Programming (FLP) for three conflicting objectives and dealt with fuzzification of objective functions. Mujumdar (2002) presented an overview of some recent mathematical tools and techniques of fuzzy optimization and fuzzy inference system for irrigation system operation, crop water allocation, and performance evaluation. Methodology for fuzzy linear programming problems was developed by Gasimov and Yenilmez (2002) and the same was used to solve the numerical example with fuzzy numbers. Raju and Duckstein (2003) proposed the Multi Objective Fuzzy Linear Programming (MOFLP) model for sustainable irrigation planning considering only the objectives fuzzy in nature and presented many advantages of the Fuzzy Linear Programming (FLP) compared with other existing multiobjective optimization methods, especially constraint and weighting methods.

Toyonaga et al. (2005) proposed a methodology and applied it to solve crop-planning problem with fuzzy random profit coefficients. Yager (1979) proposed an index to measure the membership degree of the fuzzy numbers to the fuzzy set, which is an extension of the widely accepted center of gravity defuzzification method, using goal function as weighting value. Rommelfanger (1996) presented a survey dealing with different methods to solve fuzzy linear programs. Sahoo et al. (2006) developed the linear programming and fuzzy optimization models for three conflicting objectives of irrigation planning in Mahanadi-Kathajodi delta in eastern India and compromised solution worked out for the objectives i.e. maximization of net return, crop production, and minimization of labour. Arikan and Gungor (2007) presented two-phase approach with involvement of DM, for solving the Fuzzy Parametric Programming (FPP) based on MOFLP problems by taking advantages and overcoming the disadvantages of FLP. Jimenez et al. (2007) dealt with LP problems, using various parameters as fuzzy numbers whose decision variables are crisp, and developed an interactive method for solving linear programming with fuzzy numbers. The method allows the involvement of the DM in all phases of decision process by expressing views in linguistic terms. Mangaraj and Das (2008) presented an interactive fuzzy multi-objective programming to optimize the economic and social returns for the water users, especially in the farming sector, by efficient use of water resources. In any crop planning and land use planning, where uncertainty plays an important role, a model with multiple objectives that includes
uncertainty has been developed and used (Mohaddes and Mohayidin 2008a). Mohaddes et al. (2008b) developed fuzzy multi-objective linear programming, which finds the land use optimization with social, economic, and environmental objectives.

Nasseri (2008) presented a LP problem with triangular fuzzy number and proposed new method to solve the FLP problems without any ranking function. Regulwar and Anand Raj (2008, 2009) developed a monthly Multi Objective Genetic Algorithm Fuzzy Optimization (MOGAFUOPT) model using ‘C’ language. The model has been applied to a multireservoir system in Godavari River sub basin in Maharashtra State, India and 3-D optimal surface has been developed. Zahraie and Hosseini (2010) presented an integrated approach for development of reservoir operation policies in which decision variables of the model were fuzzy coefficients of the reservoir operating rules. Regulwar and Gurav (2010) developed the MOFLP irrigation planning model that deals with fuzziness in four objective functions and applied the same to the Jayakwadi project stage-I and worked out the compromised solution under fuzzy environment. Gurav and Regulwar (2010) developed and applied the MOFLP model to Jayakwadi project stage-I, which dealt with fuzziness in the stipulations only and worked out the compromised solution. Regulwar and Gurav (2011) presented a study on irrigation planning under uncertainty considering different cases using multi objective fuzzy linear programming approach.

From the literature survey, it is seen that the irrigation planning has been carried out using fuzzy logic since more than a decade. Many of the researchers have tried and succeeded in utilizing of crisp value over fuzzy value to define various parameters of irrigation planning. The irrigation planning using FLP/MOFLP approach carried out for a number of irrigation project. The FLP/MOFLP approach for irrigation planning has been developed and applied considering fuzzy objectives and/or fuzzy resources till the date. In this study, we considered a SOFLP irrigation planning model, which had all the parameters (objective function coefficients, technological coefficients and stipulations/resources) fuzzy, except the decision variables, which were crisp. The aim of this study was to develop LP model for irrigation planning and to apply the same to find optimal cropping pattern plan with the objectives of minimization of cost of cultivation and maximization of net benefits, separately.

**METHODOLOGY**

**Description of the Study Area**

The methodology developed was applied to Jayakwadi Project Stage-I across the Godavari River, which originates from Bramhagiri Mountain at Tryambakeshwar (Maharashtra) and flows through two states, namely, Maharashtra and Andhra Pradesh. Its major tributaries are Mula, Pravara and Darana. Table 1 shows the salient features of the Jayakwadi Project Stage-I. Near the canal alignment, the soils are shallow, consisting of thin mantle of soil over the murum stratum. Deep silt and black soils are found in the adjoining area of Godavari River and its tributaries. The remaining area is occupied with soil, which is in between the above two kinds of soils. Index map of Jayakwadi Project Stage-I, is shown in Figure 1.

**Table 1. Salient Features of the Jayakwadi Project Stage-I.**

<table>
<thead>
<tr>
<th>Type of Dam</th>
<th>Earth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross capacity at FRL</td>
<td>2909 Mm³</td>
</tr>
<tr>
<td>Capacity of dead storage</td>
<td>738 Mm³</td>
</tr>
<tr>
<td>Capacity of live storage</td>
<td>2170 Mm³</td>
</tr>
<tr>
<td>Max. height of dam</td>
<td>37.73 m</td>
</tr>
<tr>
<td>Full reservoir level</td>
<td>463.906 m</td>
</tr>
<tr>
<td>Irrigable command area</td>
<td>1416.40 km²</td>
</tr>
<tr>
<td>Capacity for power generation</td>
<td>12 MW (Pumped storage plant)</td>
</tr>
</tbody>
</table>
The objectives of this study were to minimize the cost of cultivation and to maximize net benefits, separately, and to find out optimal cropping pattern for 75% dependable yield under fuzzy environment.

In the formulation of the problem, the following assumptions were made:
1. Crops considered to be grown throughout the year.
2. The irrigation intensity adopted is 22% in Kharif season (rainy/summer season), 45% in Rabi season (winter crop) and 28% in Two Seasonal, Hot Weather (HW) crop 3%, Perennial 4.5%, comprising a total irrigation intensity of 102.5%.
3. Ground water usage is not considered in the command area.
4. Only surface water has been considered for irrigation.
5. The soil of the study area is homogeneous in nature.
6. Various relationships within the models are based on the framework of linearity.
7. Same management practice has been applied to a particular crop event on each land and, hence, the cost of cultivation and net benefits under particular crop activity is constant.
8. The duration and timings of the cropping activities are considered as a constant and do not vary over years.
9. There are three seasons for growing crops viz. Kharif (April, May, June, July, August, September), Rabi (October, November, December, January, February, March) and Two Seasonal (covers period/months of Kharif and/or Rabi seasons) without any overlapping. Under certain overlapping situations, care is taken by adding specific constraints. The Hot Weather includes the months of February, March, April and May.

Objective Function

Minimization of Cost of Cultivation (CC)

See the equation (1)

\[ \text{Minimize}(CC) = \left( \sum_{i=1}^{K_i} A^K_i CC^K_i + \sum_{i=1}^{R_i} A^R_i CC^R_i + \sum_{i=1}^{TS_i} A^TS_i CC^TS_i + \sum_{i=1}^{F_i} A^F_i CC^F_i + \sum_{i=1}^{HW_i} A^HW_i CC^HW_i \right) \]

Where

\[ CC_i = [WC_i + I_{wc} + D_p + L_r + R_d + I_k + A_i + F_{lm} + F_{Ir} ] \]

\[ WC_i = [HH_{lm} + HH_{Ir} + BK_p + MC_p + S_d + M_n + Fert_n + Fert_p + Fert_k + IR_c + B_{orM_n} + P_{nc} + I_{nc} + R_n + R_m + W_c ] \]
Maximization of Net Benefits

See the Equation (2) in the below:

\[ \text{Maximize } NB = \left[ \sum_{i=1}^{K} A_i^K \text{GBC}_i^K + \sum_{i=1}^{R} A_i^R \text{GBC}_i^R + \sum_{i=1}^{TS} A_i^{TS} \text{GBC}_i^{TS} + \sum_{i=1}^{P} A_i^P \text{GBC}_i^P + \sum_{i=1}^{HW} A_i^{HW} \text{GBC}_i^{HW} \right] - \left[ \sum_{i=1}^{K} A_i^K \text{IC}_i^K + \sum_{i=1}^{R} A_i^R \text{IC}_i^R + \sum_{i=1}^{TS} A_i^{TS} \text{IC}_i^{TS} + \sum_{i=1}^{P} A_i^P \text{IC}_i^P + \sum_{i=1}^{HW} A_i^{HW} \text{IC}_i^{HW} \right] \] (2)

Constraints

Total Sowing Area Constraint

In order to take care of total area available for cultivation in command area during different crop seasons, the total area was considered as a constraint for various crops.
in the present study. The total sowing area constraint was given by,

\[
\left( \sum_{i=1}^{K} A^K_i + \sum_{i=1}^{R} A^R_i + \sum_{i=1}^{T} A^T_i + \sum_{i=1}^{P} A^P_i + \sum_{i=1}^{HW} A^{HW}_i \right) \leq CA
\]

\[
CA = \text{Total command area for all season for all crops together (ha)};
\]

**Maximum Sowing Area Constraint (According to Existing Cropping Pattern)**

The maximum sowing area constraint for various crops defined accounts for maximum sowing area available for cultivation during various crop seasons according to existing cropping pattern of the project. The maximum sowing area constraint was given by:

Kharif

\[
\left( \sum_{i=1}^{K} A^K_i + \sum_{i=1}^{R} A^R_i \right) \leq CA^K + CA^R
\]

Rabi

\[
\left( \sum_{i=1}^{R} A^R_i + \sum_{i=1}^{P} A^P_i \right) \leq CA^R + CA^P
\]

Hot Weather and Perennial

\[
\left( \sum_{i=1}^{P} A^P_i + \sum_{i=1}^{HW} A^{HW}_i \right) \leq CA^P + CA^{HW}
\]

CA^K = Command area for Kharif season for \(i^{th}\) crop (ha);

CA^R = Command area for Rabi season for \(i^{th}\) crop (ha);

CA^P = Command area under perennial crop (ha);

CA^{HW} = Command area under hot weather crop (ha).

**Affinity Constraint**

The farmers of the region have a tendency to grow cash crops and other crops according to their interest and benefits. To safeguard the interest of the food requirement of the region according to the storage capacity of the reservoir, the following limitations (upper limit using the existing cropping pattern) for various crops were incorporated as constraints,

Perennial

\[
A^P_i \leq CA^P_i
\]

A^P_i = Area under perennial crop sugarcane.

A^P_1 \leq CA^P_1

A^P_2 = Area under perennial crop banana.

Two Seasonal

A^TS_i \leq CA^TS_i

A^TS_1 = Area under two seasonal crop chilies;

A^TS_2 \leq CA^TS_2

A^TS_3 = Area under two seasonal crop L S cotton.

Kharif

A^K_1 \leq CA^K_1

A^K_5 = Area under Kharif crop sorghum.

A^K_6 \leq CA^K_6

A^K_7 = Area under Kharif crop paddy.

Rabi

A^R_1 \leq CA^R_1

A^R_2 = Area under Rabi crop sorghum.

A^R_5 \leq CA^R_5

A^R_6 = Area under Rabi crop wheat.

A^R_7 = Area under Rabi crop gram.

Hot Weather

A^{HW}_1 \leq CA^{HW}_1

A^{HW}_6 = Area under hot weather crop groundnut.

**Labour Availability**

To tackle the problem of uncertainty of availing the labour from outside the region, the labour requirement should not exceed...
Irrigation Planning with Fuzzy Parameters

the total labour availability during that interval,
Kharif
\[
\left( \sum_{i=1}^{K} A_i^K RMD_i^K + \sum_{i=1}^{P} A_i^P RMD_i^P \right) \leq LA^K + LA^P \quad (17)
\]
Rabi
\[
\left( \sum_{i=1}^{R} A_i^K RMD_i^K + \sum_{i=1}^{P} A_i^P RMD_i^P \right) \leq LA^K + LA^P \quad (18)
\]
Perennial and Hot Weather
\[
\left( \sum_{i=1}^{P} A_i^K RMD_i^K + \sum_{i=1}^{P} A_i^P RMD_i^P \right) \leq LA^K + LA^P \quad (19)
\]
\[RMD = \text{Requirement of man-days};\]
\[LA = \text{Labour availability}.
\]

Manure Availability

In order to keep the fertility of soil in rich condition, the total manure requirement should not exceed the total availability of the manure in that season.
Kharif
\[
\left( \sum_{i=1}^{K} A_i^K RMU_i^K + \sum_{i=1}^{P} A_i^P RMU_i^P \right) \leq MA^K + MA^P \quad (20)
\]
Rabi
\[
\left( \sum_{i=1}^{R} A_i^K RMU_i^K + \sum_{i=1}^{P} A_i^P RMU_i^P \right) \leq MA^K + MA^P \quad (21)
\]
Perennial and Hot Weather
\[
\left( \sum_{i=1}^{P} A_i^K RMU_i^K + \sum_{i=1}^{P} A_i^P RMU_i^P \right) \leq MA^K + MA^P \quad (22)
\]
\[RMU = \text{Requirement of manure utilization};\]
\[MA = \text{Manure availability}.
\]

Water Availability Constraint

The total water requirement of different crops should not exceed the total water availability in the reservoir.
See the Equations (23) and (24) in the below Fuzzy Linear Programming Problem with All Parameters Fuzzy in Nature
FLP model with fuzzy objective function coefficient, fuzzy technological coefficients, and fuzzy right-hand-side numbers was presented as below:
\[\text{Max } / \text{Min } = \hat{c}_j x_j \]
\[\text{s.t. } \hat{a}_{ij} x_j (\leq, \geq \hat{b}_i), \quad (i \in \mathbb{Y}_m) \quad (25)
\]
\[x_j \geq 0 (j \in \mathbb{Y}_n)
\]
Where, \(\hat{c}_j, \hat{a}_{ij}\) and \(\hat{b}_i\) are fuzzy numbers having linear membership functions and \(x_j\) are variables whose states are fuzzy numbers \((i \in \mathbb{Y}_m, j \in \mathbb{Y}_n)\); the operations of addition and multiplication are operations of fuzzy arithmetic, \(\leq\) and \(\geq\) denote the ordering of fuzzy numbers.

RESULTS AND DISCUSSION

In fuzzy linear programming, the fuzziness of available resources is characterized by the membership function over tolerance range.
In the present study, SOFLP model was
\[
\left( \sum_{i=1}^{K} A_i^K IWR_i^K + \sum_{i=1}^{P} A_i^P IWR_i^P + \sum_{i=1}^{P} A_i^{TS} IWR_i^{TS} + \sum_{i=1}^{P} A_i^{HW} IWR_i^{HW} + \sum_{i=1}^{RMD} A_i^{HW} IWR_i^{HW} \right) \leq TWA \quad (23)
\]
\[IWR = \text{Irrigation water requirement};\]
\[TWA = \text{Total water availability}.
\]
Non Negativity Constraint
\[CC, WC, I_t, D_r, I_r, R_i, A_t, F_w, F_r, WC, HH_h, HH_r, BK_r, MC_r, S_r, M_r, Fert, Fert, IR, B_r, M_r, P_r, IN, R_t, I_r, R_w, W_r, A^K, A^P, A^{TS}, A^{HW}, A_t, CA_t, CA_r, CA_r, CA_r, CA_r, CA_r, CA_r, CA_r, CA_r, CA_r, CA_r, LA^K, LA^K, LA^K, LA^K, LA^K, LA^K, LA^K, LA^K, MA^K, MA^K, MA^K, MA^K, MA^K, MA^K, PM, RMD^K, RMD^K, RMD^K, RMD^K, RMD^K, RMD^K, RMD^K, RMU^K, RMU^K, RMU^K, IWR^K, IWR^K, IWR^K, IWR^K, IWR^K, IWR^K, IWR^K, IWR^K, IWR^K, IWR^K, TWA \geq 0 \quad (24)
\]
developed and applied for the Jayakwadi Project Stage-I, Maharashtra state, India. The objectives under consideration were minimization of cost of cultivation and maximization of net benefits separately, considering various imprecise (fuzzy) parameters in the form of constraints for the existing cropping pattern of the project.

The resources/stipulations are fuzzy, as the area under irrigation can be changed based on the changes in the availability of quantity of water. In addition, the labour availability, manure availability could be changing over the entire length of the season, due to which the resources are considered as fuzzy. The technological coefficients include cost coefficients, labour requirement for crop, manure requirement for crop, water requirement for a crop; as these coefficients are also changing from sowing until the harvesting, therefore, technological coefficients are considered as fuzzy. Objective function coefficients value is also varying from sowing until the harvesting; therefore, these are taken as fuzzy.

The model for minimization of cost of cultivation [Equations (1) and (3) to (24)] so developed has been solved by finding the expected values (EV) of the various fuzzy parameters, which are in the form of triangular fuzzy numbers. Detailed procedure (Jimenez et al., 2007) for this are presented in the form of Figure 2. The findings are represented in the form of $\alpha$-acceptable optimal solutions for feasibility degree ($\alpha$= 0.4, 0.5,…, 1.0). It is also considered that the DM will not be willing to admit high risk in the violation of constraints. The DM is asked to establish an aspiration level $G$ with the help of the result obtained in Table 2. It is assumed that the DM is fully satisfied with an objective value lower than $259.89 \times 10^6$ Rs. and that he will not be able to assume a cost of cultivation more than $474.96 \times 10^6$ Rs.

In this approach, it is assumed that the feasibility degree $\alpha$, which the DM is willing to consider, is as $M= \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. It is also considered that the DM will not be willing to admit high risk in the violation of constraints. The DM is asked to establish an aspiration level $G$ with the help of the result obtained in Table 2. It is assumed that the DM is fully satisfied with an objective value lower than $259.89 \times 10^6$ Rs. and that he will not be able to assume a cost of cultivation more than $474.96 \times 10^6$ Rs. and if, for simplicity, we assume that the membership function is linear, the goal is represented by the following fuzzy subset:

$$
\mu_G(Z) = \begin{cases} 
1 & (259.89 - Z) \\
0 & (474.96 - 259.89) 
\end{cases} 
$$

$Z \leq 259.89$

$259.89 < Z < 474.96$

$Z \geq 474.96$

(26)

The compatibility index or total utility of each solution with DM’s aspiration was worked out by using the method described by Anand Raj and Nagesh Kumar (1999) as shown in Figure 3. The algorithm to work out the compatibility index or total utility values is summarized below.

1. Construct the minimizing set and maximizing set by considering the minimum and maximum values of fuzzy objective function.
2. Construct the triangular fuzzy number from the $\alpha$-acceptable optimal solutions for feasibility degree ($\alpha$= 0.4, 0.5,…, 1.0).
3. Work out utility values i.e. the membership value at the intersection...
Optimization of LP model with fuzzy parameters

\[ \text{Max} / \text{Min} = \tilde{c}_j x_j \]

s.t.

\[ \tilde{a}_{ij} x_j (\leq, \geq, \leq) \tilde{b}_i, \ (i \in \Gamma_m) \]

\[ x_j \geq 0, \ (j \in \Gamma_n) \]

(All imprecise parameters are triangular fuzzy numbers)

Work out \( \alpha \)-acceptable optimal solution with following ordinary crisp \( \alpha \)-parametric linear program.

\[ \text{Minimize} = \sum EV(\tilde{C}_j) x_j \]

s.t. \[ [(1-\alpha)E_{2ij}^L + \alpha E_{1ij}^L] \leq \alpha E_{2ij}^U + (1-\alpha)E_{1ij}^U \]

Or

\[ \text{Maximize} = \sum EV(\tilde{C}_j) x_j \]

s.t. \[ [(1-\alpha)E_{2ij}^L + \alpha E_{1ij}^L] \leq (1-\alpha)E_{2ij}^U + \alpha E_{1ij}^U \]

Interaction of DM to decide the feasibility interval (0 \( \leq \alpha \leq 1 \)) based number of linguistic labels, which he/she is able to distinguish.

Develop linear membership function with DM’s established satisfaction/aspiration level (\( Z_l \) and \( Z_u \)) of \( \tilde{G} \).

\[ \mu_{\alpha}(Z) = \begin{cases} 1 & \text{if } Z \leq Z_l \\ \frac{(Z-Z_l)}{(Z_u-Z_l)} & \text{if } Z_l < Z < Z_u \text{...Minimization} \\ 0 & \text{if } Z \geq Z_u \end{cases} \]

Or

\[ \mu_{\alpha}(Z) = \begin{cases} 1 & \text{if } Z \geq Z_u \\ \frac{(Z-Z_u)}{(Z_u-Z_l)} & \text{if } Z_l < Z < Z_u \text{...Maximization} \\ 0 & \text{if } Z \leq Z_l \end{cases} \]

Calculate the total utility / compatibility index using the following expression

\[ K_{\tilde{G}}(\tilde{z}(\alpha_k)) = U_T = (U_L + (1-U_R)/2) \]

Or

\[ K_{\tilde{G}}(\tilde{z}(\alpha_k)) = U_T = (U_R + (1-U_L)/2) \]

For each solution with DM’s aspiration using the method (Figure 3).

Using \( \tilde{b} = \tilde{r} \cap \tilde{z} \) (Bellman and Zadeh, 1970) i.e.

\[ \mu_{\alpha}(\tilde{z}(\alpha_k)) = \alpha_k = K_{\tilde{G}}(\tilde{z}(\alpha_k)) \cdot \]

Suggest the solution with highest membership degree in the fuzzy set decision.

Figure 2. Schematic representation of algorithm for minimization of cost of cultivation with all parameters as fuzzy.
Table 2. Acceptable optimal solutions for feasibility degree-\( \alpha \).

<table>
<thead>
<tr>
<th>Sr no.</th>
<th>Feasibility degree “( \alpha )”</th>
<th>Decision vector ( x^\alpha )</th>
<th>Possibility distribution of objective value, ( Z^\alpha ) = ( C_1x_1 + \ldots + C_{10}x_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>( x_1= 4882.96 ) ( x_2= 1568.62 ) ( x_3= 3209.03 ) ( x_4= 36511.99 ) ( x_5= 13541.41 ) ( x_6= 5348.38 )</td>
<td>( Z^\alpha (0.4) = (259.89 \times 10^6, 292.37 \times 10^6, 324.90 \times 10^6) )</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>( x_1= 5328.12 ) ( x_2= 1626.02 ) ( x_3= 3337.06 ) ( x_4= 39451.02 ) ( x_5= 1585 ) ( x_6= 5561.78 )</td>
<td>( Z^\alpha (0.5) = (276.78 \times 10^6, 311.37 \times 10^6, 346.02 \times 10^6) )</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>( x_1= 6339.10 ) ( x_2= 1741.51 ) ( x_3= 3597.20 ) ( x_4= 42634.57 ) ( x_5= 15451.64 ) ( x_6= 5995.34 )</td>
<td>( Z^\alpha (0.6) = (294.75 \times 10^6, 331.59 \times 10^6, 368.48 \times 10^6) )</td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>( x_1= 6914.02 ) ( x_2= 1799.62 ) ( x_3= 3729.35 ) ( x_4= 49851.96 ) ( x_5= 16117.66 ) ( x_6= 6125.58 )</td>
<td>( Z^\alpha (0.7) = (313.91 \times 10^6, 353.14 \times 10^6, 392.43 \times 10^6) )</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>( x_1= 7542.46 ) ( x_2= 1857.96 ) ( x_3= 3862.90 ) ( x_4= 53995.73 ) ( x_5= 16799.16 ) ( x_6= 6438.18 )</td>
<td>( Z^\alpha (0.8) = (334.39 \times 10^6, 376.18 \times 10^6, 418.03 \times 10^6) )</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>( x_1= 8230.95 ) ( x_2= 1916.54 ) ( x_3= 3997.90 ) ( x_4= 58445.78 ) ( x_5= 17496.71 ) ( x_6= 6633.17 )</td>
<td>( Z^\alpha (0.9) = (356.33 \times 10^6, 400.87 \times 10^6, 445.74 \times 10^6) )</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>( x_1= 8876.96 ) ( x_2= 1974.54 ) ( x_3= 4032.90 ) ( x_4= 58845.78 ) ( x_5= 18456.71 ) ( x_6= 6633.17 )</td>
<td>( Z^\alpha (1.0) = (379.92 \times 10^6, 427.40 \times 10^6, 474.96 \times 10^6) )</td>
</tr>
</tbody>
</table>

Figure 3. Utility values of fuzzy alternatives.

point, by adopting the properties of similar triangles. The left utility value will be the vertical ordinate on the horizontal line, which represents the minimum and maximum value, at the intersection point of the minimization set and the triangular fuzzy number of \( \alpha \)-acceptable optimal solutions for feasibility degree \( (\alpha = 0.4, 0.5, \ldots, 1.0) \). Similarly, work out the right utility values.

4. Calculate the total utility by using the expression \( U_T = (U_L + (1-U_R)/2) \) for
Table 3. Acceptable optimal solutions for the feasibility degree- \( \alpha \).

<table>
<thead>
<tr>
<th>Sr no.</th>
<th>Feasibility degree “( \alpha )”</th>
<th>Decision vector ( x^{i}(\alpha) )</th>
<th>Possibility Distribution of Objective Value, ( Z(\alpha) = C_{X_{1}} + \ldots + C_{X_{n}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>( X_{1} = 3466.45 ) ( X_{2} = 1683.65 ) ( X_{3} = 3464.45 ) ( X_{4} = 3759.82 ) ( X_{5} = 14800.58 )</td>
<td>( Z(0.4) = 11803.28 \times 10^{6} )</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>( X_{1} = 3377.09 ) ( X_{2} = 1626.02 ) ( X_{3} = 3351.57 ) ( X_{4} = 14164.00 ) ( X_{5} = 5561.78 )</td>
<td>( Z(0.5) = 2303.75 \times 10^{6}, 2560.16 \times 10^{6} )</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>( X_{1} = 3209.03 ) ( X_{2} = 1508.63 ) ( X_{3} = 3291.14 ) ( X_{4} = 13541.44 ) ( X_{5} = 5348.39 )</td>
<td>( Z(0.6) = 2159.22 \times 10^{6}, 2428.58 \times 10^{6} )</td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>( X_{1} = 3082.32 ) ( X_{2} = 1511.46 ) ( X_{3} = 3079.30 ) ( X_{4} = 12932.95 ) ( X_{5} = 5137.20 )</td>
<td>( Z(0.7) = 2072.50 \times 10^{6}, 2303.16 \times 10^{6} )</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>( X_{1} = 2956.92 ) ( X_{2} = 1454.53 ) ( X_{3} = 42960.66 ) ( X_{4} = 12336.39 ) ( X_{5} = 4928.20 )</td>
<td>( Z(0.8) = 1746.81 \times 10^{6}, 2183.22 \times 10^{6} )</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>( X_{1} = 2832.80 ) ( X_{2} = 1397.83 ) ( X_{3} = 2832.80 ) ( X_{4} = 11753.11 ) ( X_{5} = 4721.13 )</td>
<td>( Z(0.9) = 1654.80 \times 10^{6}, 2068.24 \times 10^{6} )</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>( X_{1} = 2709.95 ) ( X_{2} = 1341.36 ) ( X_{3} = 2709.95 ) ( X_{4} = 11182.11 ) ( X_{5} = 4516.58 )</td>
<td>( Z(1.0) = 1566.38 \times 10^{6}, 1957.73 \times 10^{6} )</td>
</tr>
</tbody>
</table>

minimization and \( U_{t} = (U_{K} + (1 - U_{L})/2 \) for maximization.

The compatibility index or total utility values are:
\[
\kappa_{6}(x^{0}(0.4)) = 0.802, \quad \kappa_{6}(x^{0}(0.5)) = 0.735, \\
\kappa_{6}(x^{0}(0.6)) = 0.640, \quad \kappa_{6}(x^{0}(0.7)) = 0.580, \\
\kappa_{6}(x^{0}(0.8)) = 0.467, \quad \kappa_{6}(x^{0}(0.9)) = 0.372, \\
\kappa_{6}(x^{0}(1.0)) = 0.272.
\]

With the help of the principle of Bellman and Zadeh (1970), if we use the t-norm algebraic product, the membership degree of each \( \alpha \)-acceptable optimal solutions to \( \tilde{B} \) (the fuzzy set that represents the balance between feasibility degree of constraints and satisfaction degree of the goal) is the following:
\[
\mu_{\tilde{B}}(x^{0}(0.4)) = 0.4(0.802) = 0.320, \\
\mu_{\tilde{B}}(x^{0}(0.5)) = 0.5(0.735) = 0.367, \\
\mu_{\tilde{B}}(x^{0}(0.6)) = 0.6(0.640) = 0.384, \\
\mu_{\tilde{B}}(x^{0}(0.7)) = 0.7(0.580) = 0.406, \\
\mu_{\tilde{B}}(x^{0}(0.8)) = 0.8(0.467) = 0.373, \\
\mu_{\tilde{B}}(x^{0}(0.9)) = 0.9(0.372) = 0.335, \\
\mu_{\tilde{B}}(x^{0}(1.0)) = 1.0(0.272) = 0.272.
\]

The solution of the fuzzy problem will be the one that has the greatest membership degree. It is found that for feasibility degree 0.7-feasible optimal solution, \( X_{1} = 6339.10, X_{2} = 1741.516, X_{3} = 3597.20, X_{4} = 46090.75, X_{5} = 15451.64, X_{6} = 13115.89, X_{7} = 20601.97, X_{8} = 40181.56, X_{9} = 5995.34, X_{10} = 3508.11 \) (\( X_{i=1}\ldots X_{j=10}: \) Area under cultivation of a particular crop \( i \) in ha). The solution has the greatest membership degree: 0.406. The results are represented graphically in Figure 4. If the DM is not satisfied with this solution, he/she can change the goal or refine the value of the different degrees of feasibility.

Similarly, the model for maximization of net benefits [(Equations (2) to (24)] was solved by evaluating the expected values (EV) of the various fuzzy parameters, which were in the form of triangular fuzzy numbers. Detailed procedure for this solution is outlined in the form of Figure 2.

The findings are represented in the form of \( \alpha \)
Figure 4. Optimal cropping pattern for minimization of cost of cultivation.

– acceptable optimal solutions as shown in Table 3.

Let us assume that the feasibility degree $\alpha$, which the Decision Maker is willing to consider, is as shown below, (We assume that the DM will not be willing to admit high risk in the violation of constraints) $M = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. The results are shown in Table 3. From these $\alpha$-acceptable optimal solutions, obtaining a better value for the optimal objective function implies a lower degree of feasibility of the constraints.

Then, the DM runs into two conflicting objectives: to improve the objective function value and to improve the degree of satisfaction of constraints. The best way to express the DM’s opinions is to express them in natural language. We established the following seven scales:

- 0.4: Completely acceptable solution;
- 0.5: Practically acceptable solution;
- 0.6: Almost acceptable solution;
- 0.7: Very acceptable solutions;
- 0.8: Quite acceptable solutions;
- 0.9: Neither acceptable nor unacceptable solutions;
- 1.0: Quite unacceptable solutions;

From the result obtained (Table 3), the DM is asked to establish an aspiration level $G$. We will suppose that the DM is fully satisfied with an objective value greater than $1566.38 \times 10^6$ Rs and that he will not be able to assume a Net Benefit more than $2698.65 \times 10^6$ Rs, and if, for simplicity, we assume that the membership function is linear, the goal will be represented by the following fuzzy subset:

$$
\mu_G(Z) = \begin{cases} 
1 & \text{if } Z > 1566.38 \\
0 & \text{if } Z \leq 1566.38
\end{cases}
$$

The compatibility index or total utility of each solution was calculated by a method suggested by Anand Raj and Nagesh Kumar (1999) as shown in Figure 3, with DM’s aspiration. The compatibility index or total utility of each solution with DM’s aspiration,
Figure 5. Optimal cropping pattern for maximization of net benefits.

According to Bellman and Zadeh (1970), if we use the t-norm algebraic product, the membership degree of each $\alpha$-acceptable optimal solutions to $\theta$ (the fuzzy set that represents the balance between feasibility degree of constraints and satisfaction degree of the goal) has the following form,

- $\mu_1(x^{\alpha}(0.4)) = 0.4(0.711) = 0.284,$
- $\mu_1(x^{\alpha}(0.5)) = 0.5(0.663) = 0.331,$
- $\mu_1(x^{\alpha}(0.6)) = 0.6(0.538) = 0.322,$
- $\mu_1(x^{\alpha}(0.7)) = 0.7(0.455) = 0.318,$
- $\mu_1(x^{\alpha}(0.8)) = 0.8(0.375) = 0.300,$
- $\mu_1(x^{\alpha}(0.9)) = 0.9(0.297) = 0.267,$
- $\mu_1(x^{\alpha}(1.0)) = 1.0(0.222) = 0.222.$

The solution of the fuzzy problem will be the one that has the greatest membership degree, 0.331: $X_1 = 3337.07$, $X_2 = 1626.02$, $X_3 = 3337.07$, $X_4 = 35175.50$, $X_5 = 14164$, $X_6 = 11803.28$, $X_7 = 18636.70$, $X_8 = 6916.81$, $X_9 = 5561.78$, $X_{10} = 3268.56$. The results are represented graphically in Figure 5. If the DM is not satisfied with this solution, he/she can change the goal or refine the value of the different degree of feasibility.

CONCLUSIONS

The Single Objective Fuzzy Linear Programming (SOFLP) model has been developed for minimization of cost of cultivation and maximization of net benefits having fuzzy parameters and applied for the Jayakwadi Project Stage-I in Godavari River sub basin in Maharashtra State, India. The proposed model is capable to tackle vagueness/uncertainty associated with the objective function coefficients, right hand side numbers/resources and technological coefficients. The observations drawn from the present study are:

The minimized cost of cultivation for irrigation planning for the present SOFLP model was found at greatest membership degree of 0.406 with the consideration of balance between the feasibility degree of
constraints and satisfaction degree of the goal.

The SOFLP model proposed had the maximized net benefits for irrigation planning at greatest membership degree of 0.331 with the consideration of balance between the feasibility degree of constraints and satisfaction degree of the goal.

The involvement of decision maker was allowed in all phases of decision-making process, which is necessary in real world problems of irrigation planning, where the data/information is vague/uncertain.

The model proposed is general-purpose model. Its usage can be practiced to the entire Godavari River basin and to the other basins with little modification related to the basin characteristics under consideration.

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پرنامه ریزی آبیاری با پارامترهای فازی: یک روش تعاملی

د. م. رگولوار و ج. پ. گوراو

چکیده

برای پیش‌سایل پرنامه ریزی آبیاری تصمیم‌های مختلف در شرایط هیدرولوژیکی تا مشخص و با ترددی جریان می‌شود و در نتیجه بی‌پیچیدگی پرنامه ریزی شیپر می‌شود. برای رفع مشکل این ترددیها می‌توان سه را به صورت پرنامه ریزی خطی فازی (FLP) در اورود. در یک خطر همین، مدل پرنامه ریزی ابیاری با پرنامه ریزی خطی فازی (SOFLP) به کار گرفته شد و هدف از آن مطالعه ورودی برابر با دست آوردو طرح کشت بهینه با کمترین هزینه کشت و کار و بیشترین منافع خالص در حالت 1 برپا کنک کرد.

در زیر-حوضه رودخانه گوداری از ابتدای ماه‌ها در هند بود. ضریب های تابع هدف، ضریب های فاکتوری، و قیمت‌های صادرات اعضا فازی متمایل در نظر گرفته شدند. برای حل مدل FLP مورد استفاده شد به این معنا که تصمیم گیری در هم‌مرحله تخمین گیری از دانلای داده (DM) به همراه استفاده که این معنا که تصمیم گیران SOFLP در تابع هدف ساده مدل دارد. نتیجه بیشتری در شرایط بیشتری مقدار جزء عضویت با حفظ تعادل بین درجه امکان پذیری محدودیت ها در درجه رضایت‌مندی هدف به دست داد. کمترین هزینه کشت و کار و بیشترین منافع خالص برای پرنامه ریزی آبیاری در مدل پیشنهادی SOFLP به ترتیب مبتنی در شرایط مقدار درجه

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عضویت برای ۱۴۰۶/۱ و ۳۳۳۴/۱ و با در نظر گرفتن تعادل بین درجه امکان پذیری محدودیت‌ها و درجه رضایتمندی هدف به دست آمد. نتیجه این‌که می‌توان تصمیم‌گیری را در همه مراحل فرایند تصمیم‌گیری دخالت داد و ابن امر در شرایط مسائل واقعی برنامه‌ریزی برای آماده‌کردن آن‌ها و اطلاعات مبهم و تردید آموده‌بی‌سیار ضروری است.