Reference Evapotranspiration Estimation Using Locally Adjusted Coefficient of Angstrom’s Radiation Model in an Arid-Cold Region

M. Raoof1*, and J. Azizi Mobaser1

ABSTRACT

Acceptable estimation of reference Evapotranspiration (ET₀) values by the Penman-Monteith FAO (PM FAO) equation requires accurate solar radiation (Rₛ) data. Rₛ values could be estimated using the Angstrom’s radiation model. The aim of this study was to determine the aₛ and bₛ coefficient (as Angstrom’s parameters) for the Ardabil plain as an arid and cold region. Angstrom’s radiation model and PM FAO equation were calibrated for the study area, by optimizing the aₛ and bₛ parameter using Generalized Reduced Gradient (GRG) method. Measured Rₛ data were collected from the Ardabil Synoptic Station and measured ET₀ data were determined using three lysimeters that were installed at the Hangar Research Station. Calibrated results showed that optimized aₛ and bₛ values were 0.117 and 0.384, respectively. Compared to the original models, errors including RMSE, AE and RE values were decreased and fitted parameters including R² and regression line slope (m) were improved in the calibrated models. The GMER values for the original models showed that Angstrom’s radiation model overestimated the Rₛ values and PM FAO equation underestimated the ET₀ values. Locally calibrated models estimated Rₛ and ET₀ values better than the original one. Nash-Sutcliffe efficiency coefficient (NSE) values proved that Rₛ and ET₀ estimation by the original models were not satisfactory, but were acceptable in the case of the calibrated models. However, calibration of Angstrom’s radiation model and PM FAO equation is necessary for each region.

Keywords: Penman-Monteith, Model calibration, Validation, Lysimeter, Ardabil

INTRODUCTION

Accurate estimates of actual evapotranspiration are a prerequisite for real-time irrigation forecasting (Doorenbos and Kassam, 1979). Lysimeters have been used to analyze precipitation (P), drainage water, root water uptake, and to determine actual evapotranspiration (ETa) (Young et al., 1996; Bakhtiari et al., 2011). Specific devices and lysimeters are required to determine evapotranspiration, various physical parameters, and soil water balance (Allen et al., 2006). The estimation of crop evapotranspiration (ETc) often involves calculating the reference evapotranspiration (ET₀) (Xu et al., 2012). Reference evapotranspiration (grass) is defined as the evapotranspiration rate from a hypothetical crop with an assumed height of 0.12 m, a fixed surface resistance of 70 s.m⁻¹, and an albedo of 0.23. The only factors affecting ET₀ are climatic parameters. Therefore, ET₀ is a climatic parameter that can be computed from weather data (Allen et al., 1998). The FAO Penman-Monteith combination equation (FAO-56 PM Equation) was proposed as a standard method for estimating reference evapotranspiration, and for evaluating other equations. It is accepted worldwide as the optimum method and the standard for evaluating other methods (e.g., Jacovides and Kontonyiannis, 1995; Antonio, 2004; Hossein et al., 2004; Xu and Chen, 2005; López- Urrea et al., 2006; Trajkovic, 2007; Meshram et al., 2010; da Silva et al., 2011; Mohawesh, 2011).

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From the original equations of Penman-Monteith and the aerodynamic and surface resistance, the FAO Penman-Monteith method to estimate ET₀ can be derived as follows (Temesgen et al., 2005):

$$ET₀ = \frac{0.408 \Delta (Rₙ-G)+\gamma \frac{900}{Tₙ+273}(eₛ-eₐ)}{\Delta+\gamma(1+0.34 u₂)}$$

(1)

Where, ET₀ is the reference evapotranspiration [mm day⁻¹], Rₙ is the net radiation at the crop surface [MJ m⁻² day⁻¹], G is the soil heat flux density [MJ m⁻² day⁻¹], T is the mean daily air temperature at 2 m height [°C], u₂ is the wind speed at 2 m height [m s⁻¹], eₛ is the saturation vapor pressure [kPa], eₐ is the actual vapor pressure [kPa], (eₛ - eₐ) is the saturation vapor pressure deficit [kPa], Δ is the slope vapor pressure curve [kPa °C⁻¹], and γ is the psychometric constant [kPa °C⁻¹].

The FAO-56 PM is a physically based approach which requires measurements of air temperature, relative humidity, solar radiation, and wind speed. Net radiation, Rₙ, is the most important parameter in the calculation of ET₀ by FAO-56 PM (Xu et al., 2009). Solar radiation, Rₛ, can be calculated with the Angstrom formula, which relates solar radiation to extraterrestrial radiation and relative sunshine duration as:

$$Rₛ = \left( aₛ + bₛ \frac{n}{N} \right) Rₐ$$

(2)

Where, Rₛ is the solar or shortwave radiation [MJ m⁻² day⁻¹], n is the actual duration of sunshine [hour], N is the maximum possible duration of sunshine or daylight hours [hour], \( \frac{n}{N} \) is the relative sunshine duration [-], Rₐ is the extraterrestrial radiation [MJ m⁻² day⁻¹], aₛ is the regression constant, expressing the fraction of extraterrestrial radiation reaching the earth on overcast days (n = 0), aₛ + bₛ is the fraction of extraterrestrial radiation reaching the earth on clear days (n = N). Depending on atmospheric conditions (humidity, dust) and solar declination (latitude and month), the Angstrom’s values aₛ and bₛ will vary. Where no actual solar radiation data are available and no calibration has been carried out for improving aₛ and bₛ parameters, the values aₛ = 0.25 and bₛ = 0.50 are recommended (Allen et al., 2006). Values for Rₐ and N for different latitudes are listed in FAO Irrigation and Drainage Paper No. 56, or could be calculated from related equations. The actual duration of sunshine, n, is recorded with a Campbell Stokes sunshine recorder. Net radiation is measured in some meteorological stations such as Ardabil station. Estimation of various Rₛ using equation (2), can affect ET₀ values change, because the hypothetical surface with uniform albedo of 0.23 can’t be realized in the measurement of net radiation in all climate stations. Based on solar radiation determined with different regimes of aₛ and bₛ, influence on calculation of ET₀ have been investigated (Xu et al., 2006). Xu et al. (2009) concluded that errors in ET₀ caused by different values of aₛ and bₛ are not neglectable. In stations with solar radiation measurement, it is possible and necessary to evaluate and estimate the Angstrom’s coefficients, to calculate solar radiation and hence ET₀. Many studies have been done on the estimation of reference plant evapotranspiration by various methods. Gocic’ et al. (2015) indicated that SVM-W (support vector machine–wavelet) is the best methodology for prediction of ET₀, whereas SVM–Wavelet and SVM-FFA (support vector machine–firefly algorithm) models have higher correlation coefficient as compared to ANN (artificial neural network) and GP (genetic programming) computational methods. Petkovic’ et al. (2015) used the neuro-fuzzy inference system (ANFIS) for selection of the most influential reference evapotranspiration (ET₀) parameters. They concluded that, among the input variables, sunshine hours, actual vapor pressure, and minimum air temperature are the most influential for ET₀ estimation. In another research, Petkovic’ et al. (2016) showed that the radial basis function network with particle swarm optimization (RBFN-PSO) had better
statistical characteristics than radial basis function network with back propagation (RBFN-BP) and could be helpful for the ET₀ estimation. Shamshirband et al. (2016) indicated that combination of adaptive neuro fuzzy interference system (ANFIS) and cuckoo search algorithm (CSA) could be used for ET₀ estimation with high reliability. Gocic’ et al. (2016) used the extreme learning machine (ELM) for estimating monthly reference evapotranspiration (ET₀) in two weather stations including Nis and Belgrade, Serbia. Results showed that adjusted Hargreaves model was found to be superior in modeling monthly ET₀ than the Priestley-Taylor and Turc models.

The main aim of this study was to investigate the effect of different values of Angstrom’s coefficients for estimation of solar radiation and ET₀ in Ardabil plain as an arid and cold region, using lysimeter data.

MATERIALS AND METHODS

Site Description

This study was conducted in Ardabil plain (38°, 10’ to 38°, 15’ N, 48°, 15’ to 48°, 20’ E and 1350 m elevation above the sea). The Ardabil climate station (38°, 15’ N, 48°, 17’ E) was selected as a typical climate station in an arid and cold region in northeast of Iran (Figure 1). Experimental site was located at Hangar Farm of Mohaghegh Ardabili University. The annual mean precipitation rate, average minimum monthly temperature, and average maximum monthly temperature through 1995 to 2015 were obtained as 280.9 mm, 2.4 °C, and 15.07 °C, respectively.

For the following two reasons, the length of the study period was considered 81 days. 1. The growth period of the grass is 45 days and then the cutting period. Thus, the earliest 45 days were the growth period, and the end 36 days were the cutting period. 2. Snow in early November (Aban) led to the completion of the study period.

At the study period, the averages of minimum (Tmin) and maximum air temperature (Tmax), daily sunshine hour (n), actual relative humidity (RHₐ), solar radiation (Rₛ), and wind speed (v) were 10.43 °C, 25.04 °C, 8.85 hr, 65.31%, 5.25 mm/day and 3.33 m/s, respectively. Statistical parameters of the observed meteorological data from 22 July, 2014, to 10 October, 2014, (calibration and validation periods) are listed in Table 1.

Figure 1. Location of study area.
Lysimeters Characteristics

Three lysimeters were installed to determine the water balance equation, estimate reference evapotranspiration, and calibrate Angstrom’s Radiation Model (determining the $a$ and $b$ coefficients) in Hangar Farm site. Sectional drawing of a lysimeter is illustrated in Figure 2. Dimensions of each lysimeter were 60 cm diameter and 90 cm height. To measure the water content changes in each lysimeter, 6 sensors (gypsum block) were installed at depth of 5, 15, 25, 40, 60 and 80 cm, from the soil surface. To measure soil moisture, it is necessary that the resistance of the gypsum blocks is measured, then, the values of the block resistance are converted to soil moisture using a calibration curve. Blocks resistance data was measured at 10:00 AM and 18:00 PM every day using an ELE-MC-302 soil moisture instrument. Drained water was collected in a drainage tank located under the lysimeters and was measured two times every day, the same as blocks resistance.

In the lysimeters, grass was planted, as reference crop, on 22 July, 2015, and irrigated every 3 days, according to the lysimeters soil type. Given that the purpose of this study was to measure potential evapotranspiration, the irrigation interval was chosen to avoid plant stress. For irrigation intervals of 3 days, soil type did not create restrictions.

To calculate soil water storage, lysimeters soil was divided into 6 layers, including the depth of 0-10, 10-20, 20-32.5, 32.5-50, 50-70, and 70-90 cm, according to the depth of installation of gypsum blocks (at depths of 5, 15, 25, 40, 60 and 80 cm). The value of the soil water content was obtained for each of the layers using the gypsum blocks data. To determine some soil physical and hydraulic properties, three disturbed and three undisturbed soil sample were taken. The total soil water storage was determined by the sum of storage in each of the considered layers (Equation 3) (Feltrin et al., 2011):

$$S = \int_{0}^{L} \theta \, dz \approx \sum_{i=1}^{n} \theta \Delta z = \theta L \quad (3)$$

Where, $S$ is the soil water storage (mm), $\theta$ is the volumetric soil water content ($\text{cm}^3/\text{cm}^3$), $L$
is the total soil depth (mm), and n is the number of layer. Thus, the change in soil water storage was determined by the difference between the values of the soil water content obtained in the final and initial time of each period (daily period), using equation 4:

$$\Delta S = S_f - S_i$$ (4)

Where, $\Delta S$ is the change in soil water storage (mm), $S_i$ and $S_f$ are the initial and final soil water storage (mm), respectively. The water balance equation of the lysimeters was used to calculate the grass evapotranspiration ($ET_0$). The evapotranspiration was obtained by the difference between the soil water inputs and outputs (Equation 5):

$$ET = P - D + \Delta S$$ (5)

Where, ET is the evapotranspiration (mm), P is the rainfall (mm), D is the drainage (mm), and $\Delta S$ is the change in soil water storage (mm).

**PM FAO Equation: Calibration and Validation**

The coefficients $a_i$ and $b_i$, with the original values of 0.25 and 0.5, respectively, should be determined according to the local calibration. Based on observed evapotranspiration data from 22 July to 13 September, 2014, local calibration was performed to determine the values of coefficients $a_i$ and $b_i$ through nonlinear multiple regression for the $ET_0$ calculated using the FAO-56 PM equation and measured $ET_0$. The nonlinear multiple regressions were realized using the Solver extension of Excel software with Generalized Reduced Gradient method (GRG). The locally calibrated PM FAO equation was validated for the data from 14 September to 10 October 2014, by comparing the results with the measured data. For evaluation of the $ET_0$ calculated by PM FAO equation relative to the measured data, firstly, the calculated and measured data were plotted around the 1:1 line. Linear regressions with zero interception were made, and slopes and determination coefficients ($R^2$) were calculated.

**Statistical Analysis**

In the present study, root mean square error (RMSE), average absolute errors (AE), Nash-Sutcliffe efficiency coefficient (NSE) and relative error (RE), were used to evaluate the Angstrom’s radiation model (in estimation of $R_i$) and PM FAO equation (in estimation of $ET_0$), derived from different values of $a_i$ and $b_i$ coefficient. Statistical parameters were calculated using the following expressions:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (O_i - P_i)^2}$$ (6)

$$AE = \frac{1}{n} \sum_{i=1}^{n} |O_i - P_i|$$ (7)

$$NSE = 1 - \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2}$$ (8)

$$RE = \frac{\sum_{i=1}^{n} |O_i - P_i|}{\sum_{i=1}^{n} O_i} \times 100$$ (9)

Results were categorized into four groups by analysis of the relative error using the following criteria (Xu et al., 2009):

- RE <10% indicates very good,
- 15%<RE<10% indicates good,
- 20%<RE<15% indicates acceptable, and
- RE>20% indicates poor result.

To evaluate the overestimation or underestimation of Angstrom’s radiation model and PM FAO equation, geometric mean error ratio (GMER) were also used as follows:

$$GMER = \exp\left(\frac{1}{n} \sum_{i=1}^{n} \ln\left(\frac{O_i}{P_i}\right)\right)$$ (10)

If predicted values were equal to observed values, GMER reach unity. A GMER>1 indicates overestimation and GMER<1 shows underestimation of the mentioned values (Wagner et al., 2001). In all evaluation equations (Equations 6 to 10), $O_i$ and $P_i$ represent the observed and predicted data, respectively.

**RESULTS AND DISCUSSION**

**Soil Properties and Water Balance**

Results from the granulometric analysis of the samples collected at the site of the lysimeters installation are shown in Table 2. The amount of sand percentage, which was over 60% of the total in all lysimeters, caused soil available water to decrease and drained
water to increase. Some soil hydraulic properties that determine the soil available water are illustrated in Table 3. Total soil available water was 140 mm/m, as calculated from the field capacity and permanent wilting point water content. Volumetric soil water content changes in depth of 5, 15, 25, 40, 60 and 80 cm and water storage in the period are shown in Figure 3. In this figure, increase in water content indicates irrigation (wetting trend) and decrease in the water content indicates soil water extraction (drying trend). Because evaporation is more intense in the surface layer, for the depth of 5 and 15 cm, water content changes more rapidly than other depths. The positive and negative values of water storage represent the frequent addition and depletion of water in the lysimeters.

Evaluation of Estimated $R_s$ and $ET_0$ during the Entire Period

Daily radiation values estimated by the Angstrom’s radiation model and daily $ET_0$ values estimated by the PM FAO equation compared with the measured data are illustrated in Figures 4 and 5, respectively, in which $R_s$ and $ET_0$ values are also shown around the 1:1 (one to one) line. Regression equation has been derived between the measured and estimated $R_s$ and $ET_0$ with zero interception. The radiation values in warm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Particle Density (gr/m³)</th>
<th>Bulk density (gr/m³)</th>
<th>Total Porosity (%)</th>
<th>Clay (%)</th>
<th>Silt (%)</th>
<th>Sand (%)</th>
<th>Texture Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.4235</td>
<td>1.225</td>
<td>49.45</td>
<td>6.67</td>
<td>31.47</td>
<td>61.87</td>
<td>Sandy</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0456</td>
<td>0.0925</td>
<td>3.203</td>
<td>0.9428</td>
<td>2.494</td>
<td>2.944</td>
<td></td>
</tr>
<tr>
<td>CV</td>
<td>0.02</td>
<td>0.07</td>
<td>0.06</td>
<td>0.14</td>
<td>0.08</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$θ_r$ (cm³/cm³)</th>
<th>$θ_{FC}$ (cm³/cm³)</th>
<th>$θ_s$ (cm³/cm³)</th>
<th>$K_s$ (cm/day)</th>
<th>α (-)</th>
<th>n (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1226</td>
<td>0.2626</td>
<td>0.4945</td>
<td>281.6</td>
<td>0.01556</td>
<td>1.76466</td>
</tr>
</tbody>
</table>

**Figure 3.** Volumetric soil water content changes in depth of 5, 15, 25, 40, 60 and 80 cm and daily water storage.
days are greater than cold days. According to Figures 4 and 5, measured daily radiation values fluctuated between 0.81 and 7.63 mm/day and measured evapotranspiration (ET₀) values fluctuated between 0.54 and 12.42 mm/day. Before local calibration and validation (with \( a_s = 0.25 \) and \( b_s = 0.75 \), recommended by FAO), the PM FAO model had a high error and therefore the model must be calibrated and validated locally. Statistical parameters of Angstrom’s radiation model and PM FAO equation estimation are showed in Table 4. In both \( R_s \) and ET₀ estimation with \( a_s \) and \( b_s \) coefficient recommended by FAO, results are not satisfactory. The RMSE, AE, and RE, which explain the error between the measured and estimated parameters, have high values, indicating that Angstrom’s radiation model and PM FAO equation require calibration in each region.

![Figure 4](image1.png)

**Figure 4.** Daily radiation values (\( R_s \)) estimated by the Angstrom’s radiation model compared with the measured data, before local calibration (during the entire period and around the 1:1 line)

![Figure 5](image2.png)

**Figure 5.** Daily ET₀ values estimated by the PM FAO equation compared with the measured data, before local calibration (during the entire period and around the 1:1 line)

**Table 4.** Statistical parameters of Angstrom’s radiation model and PM FAO equation before local calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.861</td>
<td>Acceptable</td>
<td>( R^2 )</td>
<td>0.27</td>
<td>Non acceptable</td>
</tr>
<tr>
<td>M (line slope)</td>
<td>1.475</td>
<td>Non acceptable</td>
<td>M (line slope)</td>
<td>0.864</td>
<td>Non acceptable</td>
</tr>
<tr>
<td>RMSE (mm/day)</td>
<td>2.882</td>
<td>High</td>
<td>RMSE (mm/day)</td>
<td>1.778</td>
<td>High</td>
</tr>
<tr>
<td>AE (mm/day)</td>
<td>2.688</td>
<td>High</td>
<td>AE (mm/day)</td>
<td>1.342</td>
<td>High</td>
</tr>
<tr>
<td>NSE (-)</td>
<td>-0.114</td>
<td>Non satisfied</td>
<td>NSE (-)</td>
<td>0.291</td>
<td>Non satisfied</td>
</tr>
<tr>
<td>RE (%)</td>
<td>49.061</td>
<td>Poor</td>
<td>RE (%)</td>
<td>30.96</td>
<td>Poor</td>
</tr>
<tr>
<td>GMER (-)</td>
<td>0.656</td>
<td>Overestimated</td>
<td>GMER (-)</td>
<td>1.054</td>
<td>Underestimated</td>
</tr>
</tbody>
</table>
Value of $R^2$ is acceptable for radiation model, but it is small for PM FAO equation. For $ET_0$ values, points are very far from 1:1 line. In Table 4, $M$ values represent the slope of the fitted line between the measured and estimated data with zero interception. The value of $M$ close to one indicates high accuracy of the model. For both models, Nash-Sutcliffe efficiency coefficient (NSE) values are not acceptable, because NSE of both models is less than 0.36. This parameter is better for $ET_0$ estimation than for $R_e$ estimation. The GMER values showed that the Angstrom’s radiation model overestimated the $R_e$ values, but PM FAO equation underestimated the $ET_0$ values. $R_e$ overestimation and $ET_0$ underestimation related to parameters of those equations. In the study period, the average of overestimation of $R_e$ is 2.69 mm/day, corresponding to a relative difference of approximately 49.06% with respect to measured $R_e$ data. The average of underestimation of $ET_0$ is 0.29 mm/day, corresponding to a relative difference of approximately 6.66% with respect to measured $ET_0$ data. These overestimation and underestimation could not be ignored.

Local Calibration and Validation

Local calibration of Angstrom’s radiation model and then PM FAO equation was carried out using two third of the entire data, including 54 days. Calibration was performed by optimization of Angstrom’s $a_e$ and $b_e$ coefficients using the Solver extension of Excel software with Generalized Reduced Gradient (GRG) method. Local $a_e$ and $b_e$ coefficients were obtained equal to 0.117 and 0.384, respectively. To validate the models, values of $R_e$ and $ET_0$ were estimated using calibrated radiation and $ET_0$ equation for one third of the entire data, including 27 days. Coefficients $a_e$ and $b_e$ in the current study are much smaller than the original value suggested by Allen et al. (2006) and Xu et al. (2009).

Evaluation of Estimated $R_e$ and $ET_0$ during the Calibration and Validation Periods

Coefficients $a_e$ and $b_e$ were estimated in Ardabil plain as an arid-cold region to improve results of Angstrom’s radiation model and PM FAO equation for estimation of $R_e$ and $ET_0$. In the calibration step, the values of $a_e$ and $b_e$ were determined as 0.117 and 0.384 for the study region. The $R_e$ and $ET_0$ values were re-calculated with these new values of $a_e$ and $b_e$. For calibration step, estimated daily radiation values compared with those measured one are illustrated in Figure 6. Estimated daily $ET_0$ values compared with the measured data, also, are illustrated in Figure 7. The statistical parameters of the Angstrom’s radiation model and PM FAO equation, related to the calibration step, are given in Table 5.

Validation of the model was carried out with 27 data. The values obtained from the calibration step, were used in the validation step. The values of $ET_0$ and $R_e$ obtained in the validation step are shown in Figures 8 and 9, respectively. The statistical parameters of the Angstrom’s radiation model and PM FAO equation, related to the validation step, are also given in Table 6. Comparing the results of the original Angstrom’s radiation model and original PM FAO equation with those locally calibrated showed that the errors (RMSE, AE and RE) were decreased and the fitted parameters ($M$, $R^2$ and NSE) were improved. In estimation of $R_e$, the RMSE, AE and RE decreased 2.427 mm/day, 2.381 mm/day and 42.041%, respectively. In estimation of $ET_0$, the RMSE, AE and RE also decreased 1.01 mm/day, 0.895 mm/day and 16.246%, respectively.

The $R^2$ values in validation step were increased compared to before calibration from 0.861 to 0935 for $R_e$ model and from 0.27 to 0.877 for $ET_0$ equation, indicating that the $ET_0$ estimation improved highly. The GMER values in different stages proved that underestimation or overestimation of $R_e$ and $ET_0$ decreased in locally calibrated models.
and caused the line slopes (M) reach near unity and the points were perched in around the 1:1 line in Figures 6 to 9. If NSE values reach values more than 0.36, the efficiency of the model will be satisfactory. In locally calibrated models, compared to the original models, the NSE values were increased from -0.114 to 0.887 in Rs model and from 0.291 to 0.809 in $ET_0$ model. The RE value and its criteria showed that the original Angstrom’s radiation model and the original PM FAO equation have poor accuracy, whereas the locally calibrated ones have very good and good accuracy, respectively.

**CONCLUSION**

For an arid-cold region, the Angstrom’s radiation model and the PM FAO equation

![Graph 6](image6.png)

**Figure 6.** Daily Rs values estimated by the locally calibrated Angstrom’s radiation model compared with the measured data (during the calibration period and around the 1:1 line)

![Graph 7](image7.png)

**Figure 7.** Daily $ET_0$ values estimated by the locally calibrated PM FAO equation compared with the measured data (during the calibration period and around the 1:1 line)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$ (-)</td>
<td>0.922</td>
<td>Acceptable</td>
<td>$R^2$ (-)</td>
<td>0.881</td>
<td>Acceptable</td>
</tr>
<tr>
<td>M (line slope) (-)</td>
<td>0.973</td>
<td>Acceptable</td>
<td>M (line slope) (-)</td>
<td>0.903</td>
<td>Acceptable</td>
</tr>
<tr>
<td>RMSE (mm/day)</td>
<td>0.483</td>
<td>Low</td>
<td>RMSE (mm/day)</td>
<td>0.818</td>
<td>Low</td>
</tr>
<tr>
<td>AE (mm/day)</td>
<td>0.215</td>
<td>Low</td>
<td>AE (mm/day)</td>
<td>0.447</td>
<td>Low</td>
</tr>
<tr>
<td>NSE (-)</td>
<td>0.891</td>
<td>Satisfied</td>
<td>NSE (-)</td>
<td>0.838</td>
<td>Satisfied</td>
</tr>
<tr>
<td>RE (%)</td>
<td>3.483</td>
<td>Very good</td>
<td>RE (%)</td>
<td>9.183</td>
<td>Very good</td>
</tr>
<tr>
<td>GMER (-)</td>
<td>1.045</td>
<td>Underestimated</td>
<td>GMER (-)</td>
<td>1.101</td>
<td>Underestimated</td>
</tr>
</tbody>
</table>
with original recommended angstrom’s coefficients were evaluated. The capabilities to estimate Rs and ET₀ values were examined using some statistical parameters. Both original models had low accuracy in estimation of Rs and ET₀ values. Originals Angstrom’s radiation model overestimated and original PM FAO equation underestimated the values of daily Rs and ET₀, respectively. Therefore, local calibration of both models was performed in Ardabil plain using the measured Rs and ET₀ values. Three lysimeters were installed at the study site to measure ET₀ values. To optimize the aᵣ and bᵣ parameters, the GRG optimization method from the Solver extension of Excel

**Table 6. Statistical parameters of locally calibrated Angstrom’s radiation model and PM FAO equation in the validation stage.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
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<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angstrom’s radiation model</td>
<td></td>
<td></td>
<td>PM FAO equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R² (-)</td>
<td>0.935</td>
<td>Acceptable</td>
<td>R² (-)</td>
<td>0.877</td>
<td>Acceptable</td>
</tr>
<tr>
<td>M (line slope) (-)</td>
<td>0.932</td>
<td>Acceptable</td>
<td>M (line slope) (-)</td>
<td>0.852</td>
<td>Acceptable</td>
</tr>
<tr>
<td>RMSE (mm/day)</td>
<td>0.455</td>
<td>Low</td>
<td>RMSE (mm/day)</td>
<td>0.768</td>
<td>Low</td>
</tr>
<tr>
<td>AE (mm/day)</td>
<td>0.287</td>
<td>Low</td>
<td>AE (mm/day)</td>
<td>0.447</td>
<td>Low</td>
</tr>
<tr>
<td>NSE (-)</td>
<td>0.887</td>
<td>Satisfied</td>
<td>NSE (-)</td>
<td>0.809</td>
<td>Satisfied</td>
</tr>
<tr>
<td>RE (%)</td>
<td>7.02</td>
<td>Very good</td>
<td>RE (%)</td>
<td>14.714</td>
<td>Good</td>
</tr>
<tr>
<td>GMER (-)</td>
<td>1.083</td>
<td>Underestimated</td>
<td>GMER (-)</td>
<td>1.171</td>
<td>Underestimated</td>
</tr>
</tbody>
</table>

**Figure 8.** Comparing the estimated Rs values with the measured data in validating the locally calibrated Angstrom’s radiation model

**Figure 9.** Comparing the estimated ET₀ values with the measured data in validating the locally calibrated PM FAO equation
software was used. To calibrate the Angstrom’s radiation model and PM FAO equation, a and b coefficients were determined by using the Solver extension of Excel software. The values of a and b were determined as 0.117 and 0.384 for the study region (compared to 0.25 and 0.5 as original values). The errors of Angstrom’s radiation model and PM FAO equation were decreased and the fitted parameters were increased by local calibration of the models. By using the new values of a and b (using calibrated models), the estimates for R, and PM FAO models were decreased and the accuracy of the models improved and the accuracy of the models increased. Therefore, the calibration of R, and PM FAO models is essential for each region.

**ACKNOWLEDGMENT**

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**REFERENCES**


تخمین تبخیرتعرق مرجع با استفاده از ضرایب تعدل شده محیط معادله تابش آنگستروم
در یک منطقه سرد و خشک
م. رئوف و ج. عزیزی مبصر

چکیده
تخمین قابل قبول تبخیرتعرق مرجع (ET0) بوسیله رابطه پنمن مونتیث فائو به کمک معادله تابش آنگستروم زده شود. هدف اصلی این مطالعه تعیین ضرایب a (sR) و b (PM FAO) نقاط سرد و خشک می باشند. مدل تابش آنگستروم و معادله پنمن مونتیث فائو با بهینه کردن مقادیر a و b با استفاده از روش گرادیان کاهشی تعیین شده است. برای منطقه مورد مطالعه و استنیج گردن. مقادیر a و b با استفاده از روش گرادیان کاهشی تعیین شده است. برای منطقه مورد مطالعه و استنیج 

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با استفاده از سه عدد لاپسیمتر که در استحکام محاسباتی قرار گرفته بود، تعیین شد. نتایج واسنجی مدل‌های نشان داده که مقادیر بهینه شده بهترین پاسخ آن‌ها به پیش‌بینی‌های اصلی مقدار حداکثر خطا شامل هستند. مقادیر پارامترهای برآورد شده شامل $R^2$ و شبکه خط رگرسیون بهبود یافته از $R_s$ و مقادیر پارامترهای برآورد و مدل پیشنهادی مقدارهای اصلی نشان داد که مدل تابش آنگسترورم $R_s$ را برآورد کرده است. مدل پیشنهادی مقدارهای اصلی $R_s$ و $ET_0$ با استفاده از مقادیر پارامترهای برآورد، مقادیر محاسبه شده از $R_s$ و $ET_0$ با استفاده از مقادیر محاسبه شده از $R_s$ و $ET_0$ بدین ترتیب واسنجی مدل تابش آنگسترورم و معادله پنمن مونتیث فائو برای هر منطقه‌ای ضروری می‌باشد.